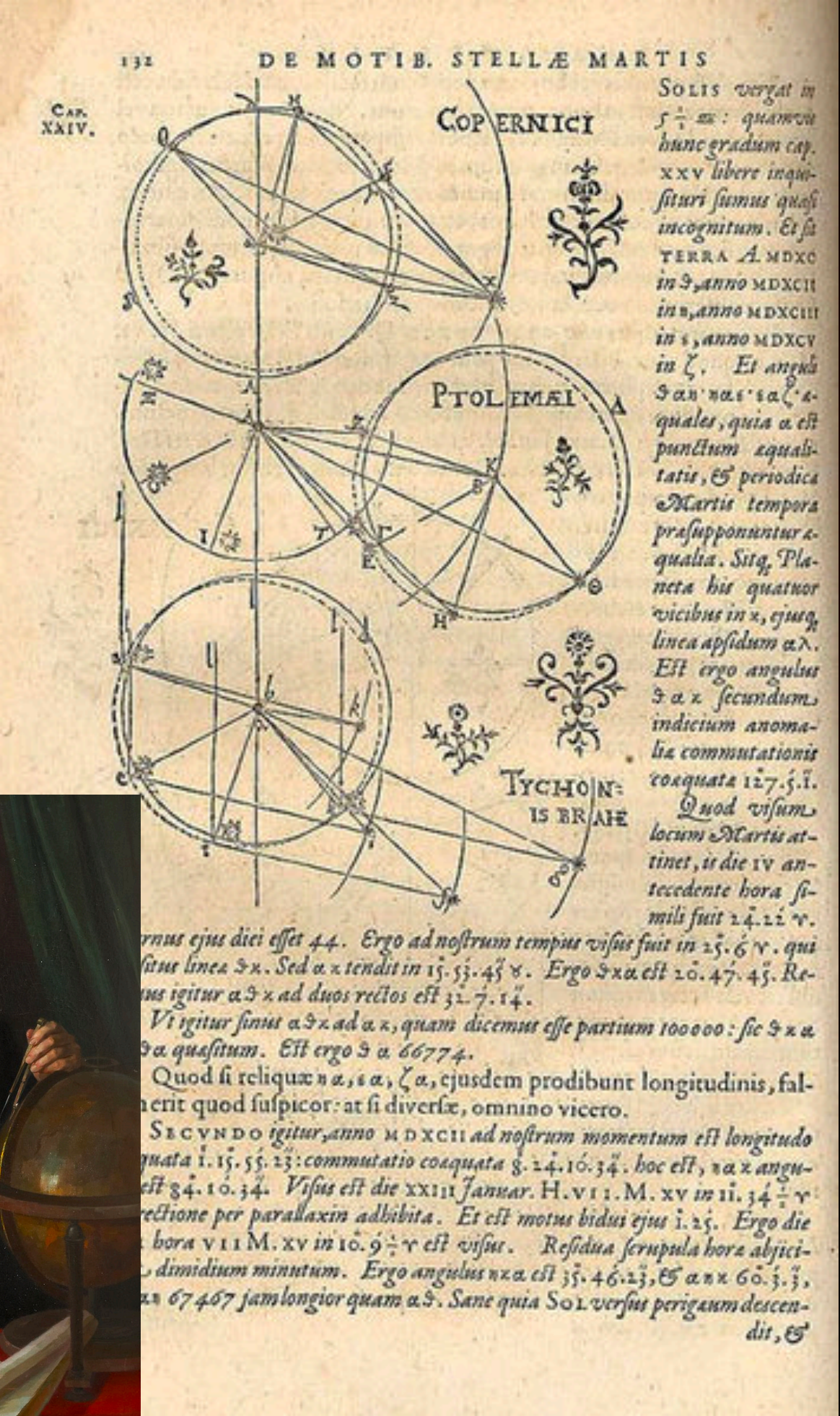
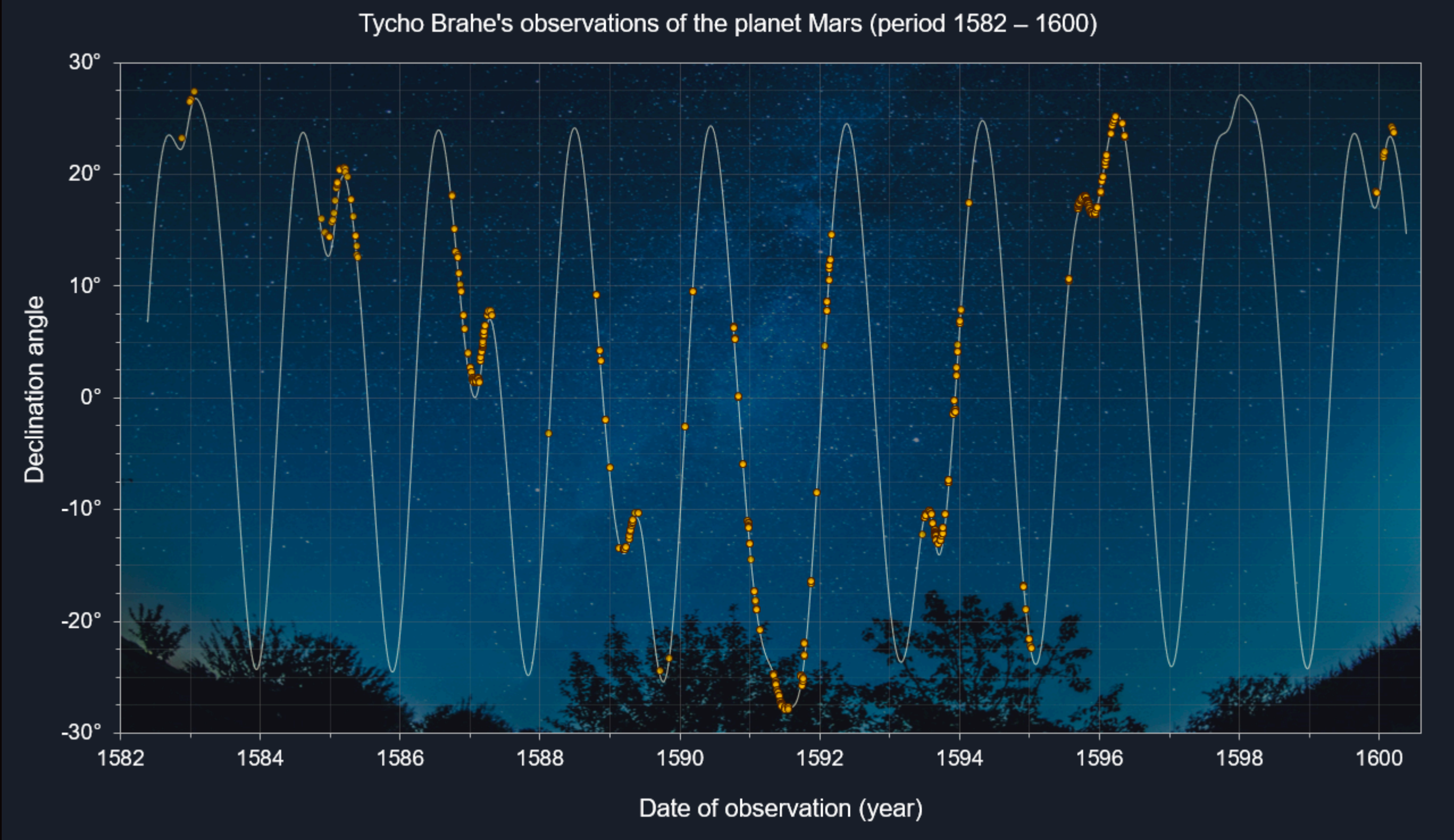


YY Ahn,
Indiana University

Imagining the Space of Knowledge

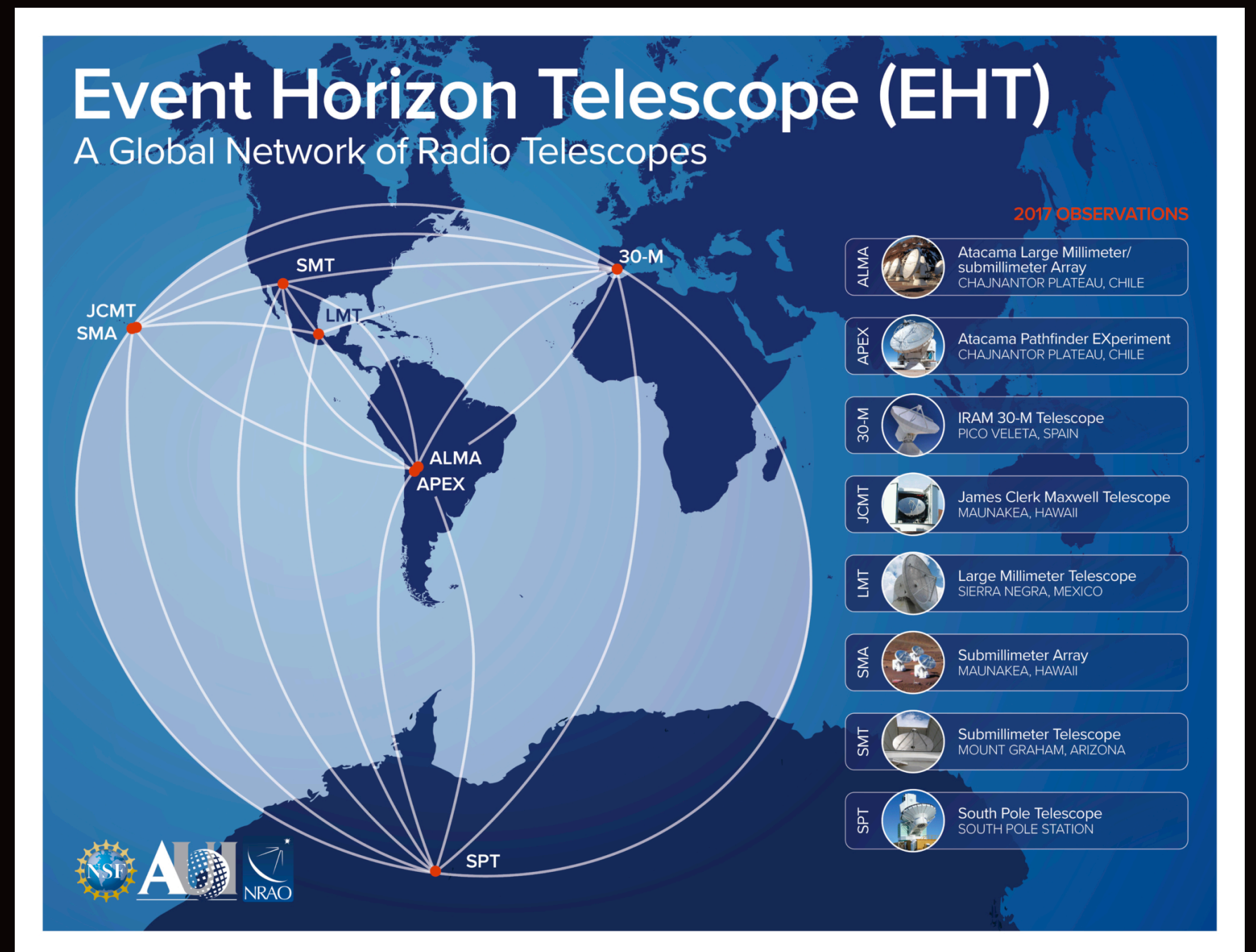
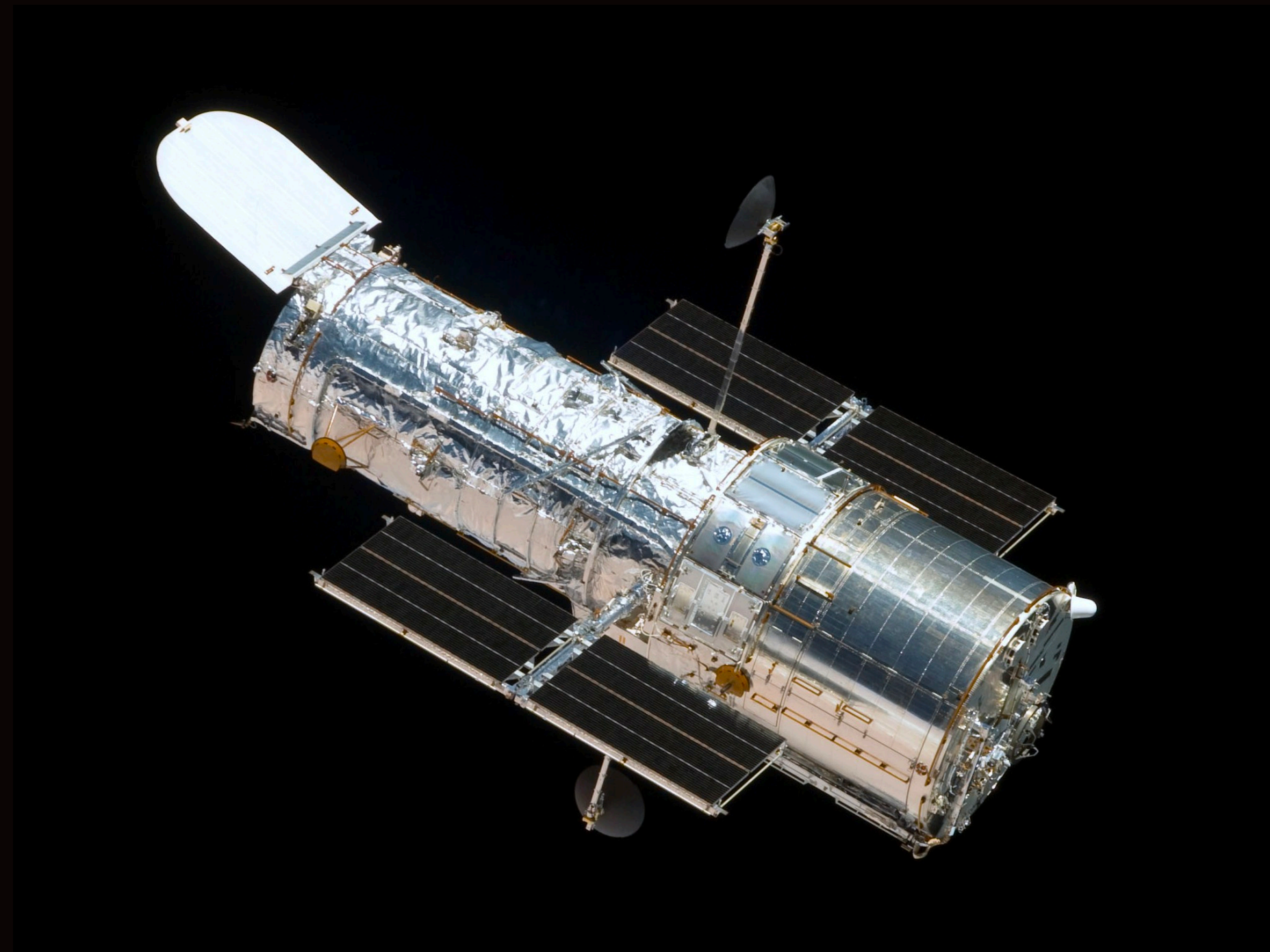


Observation and measurement are the bedrock of science



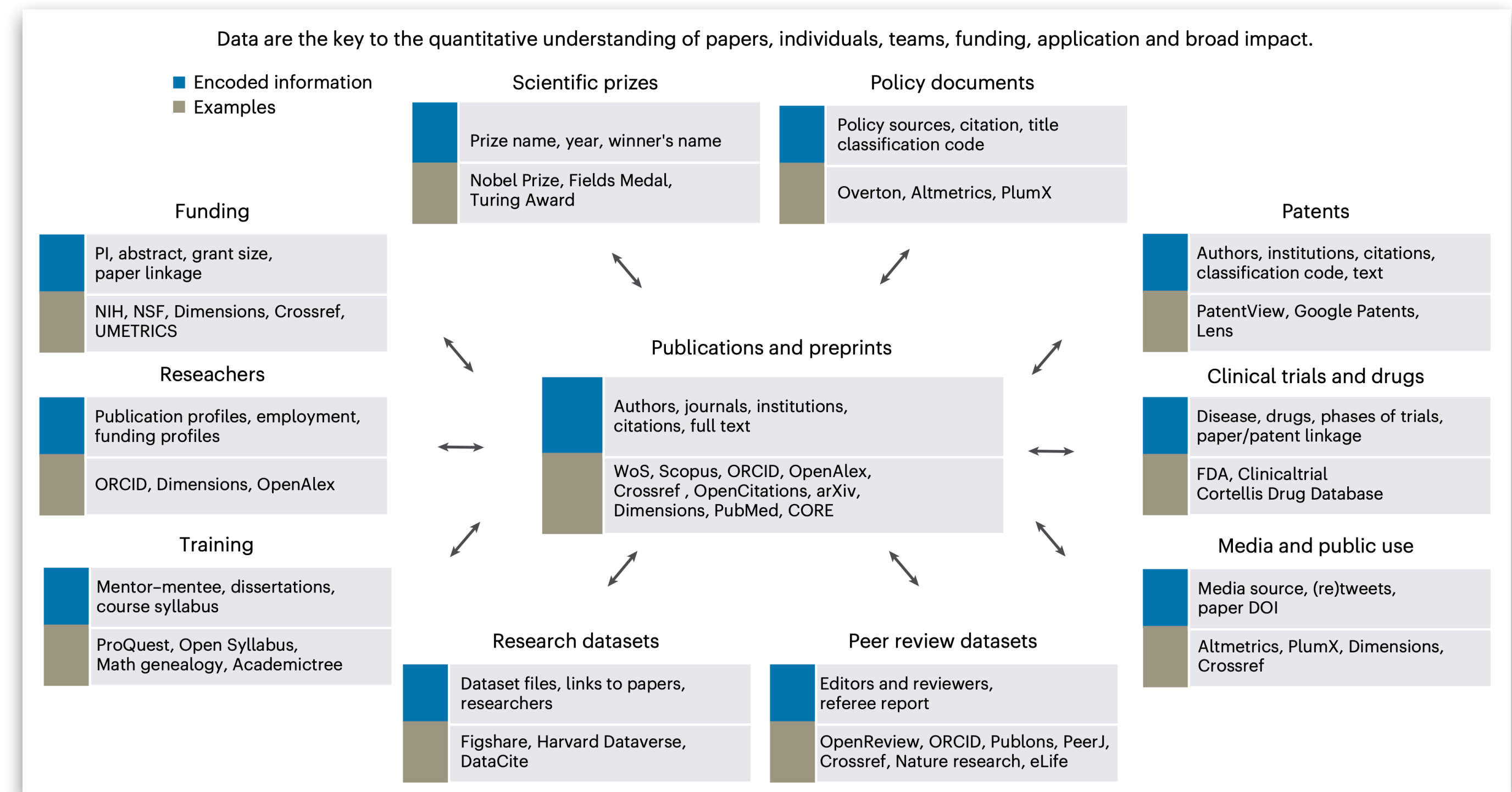
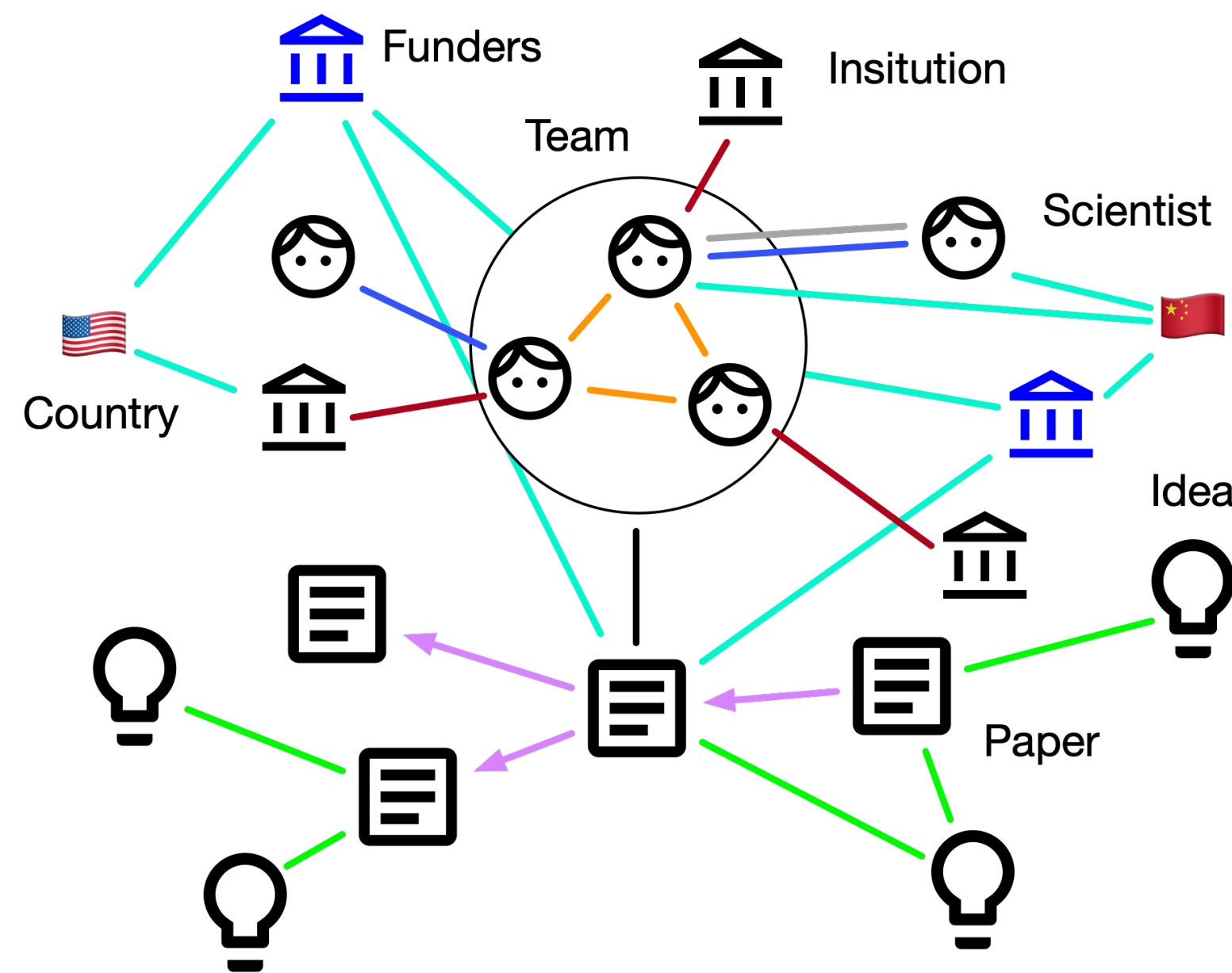
<https://observablehq.com/@christophe-yamahata/visualizing-tycho-brahe-s-astronomical-observations-mars>

Instruments matter



**What could be the EHT/JWT
for Science of Science?**

More data is becoming available



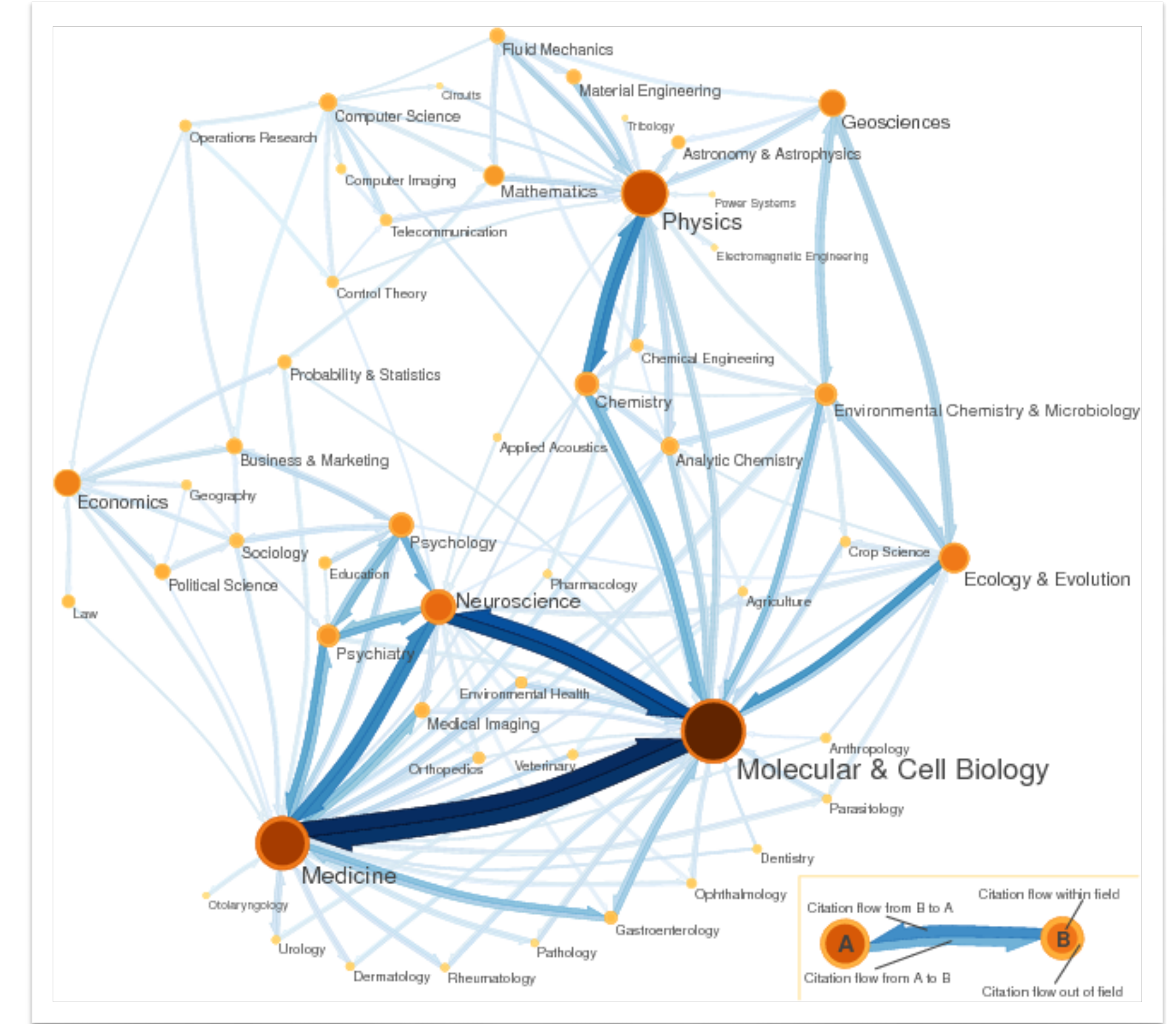
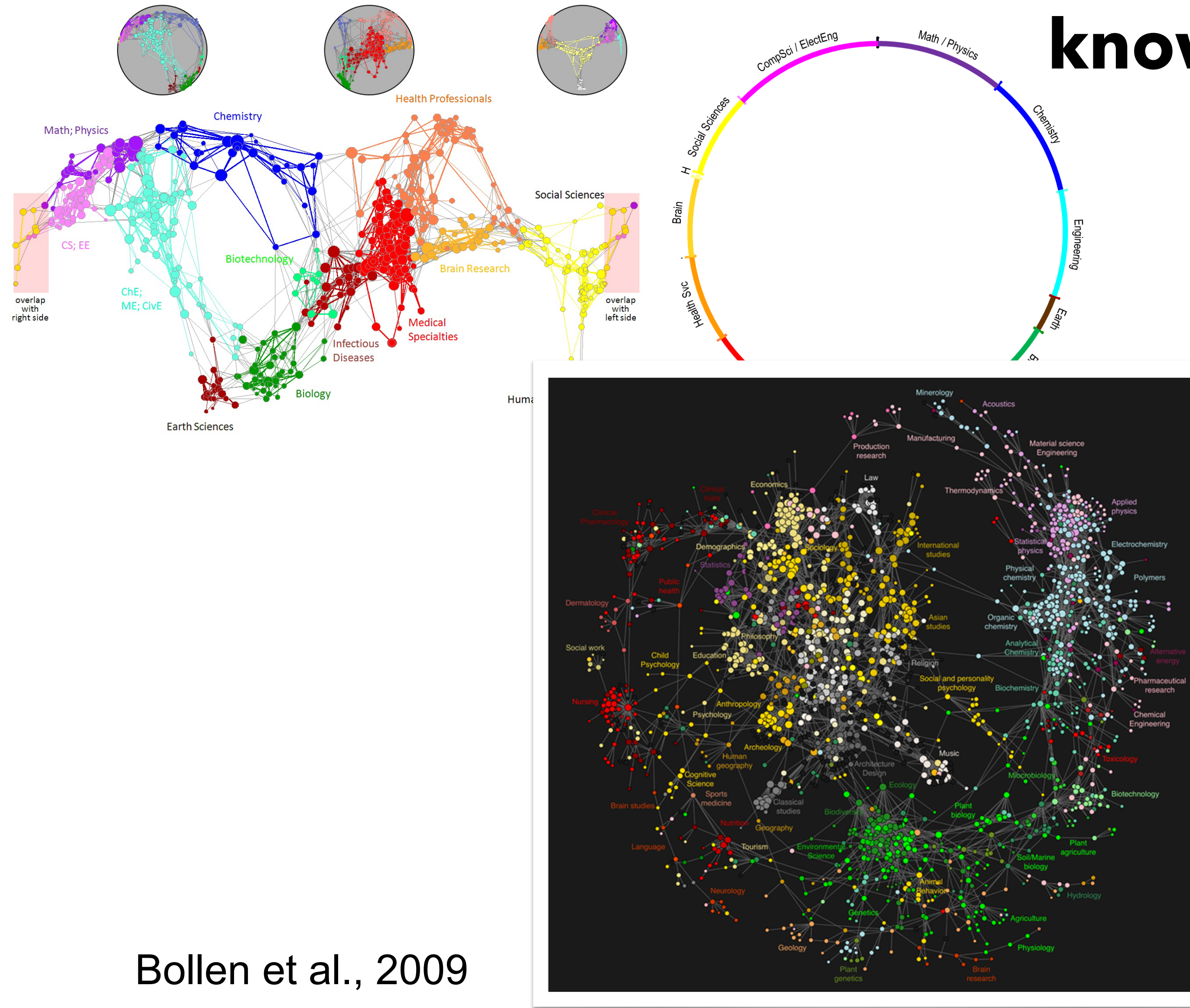
Liu, Lu, et al. "Data, measurement and empirical methods in the science of science." *Nature Human Behaviour* (2023): 1-13.

But they are complex, heterogeneous, and interconnected.



Börner, Klavans, Boyak, et al., 2012

Maps of Science—imagining the knowledge space



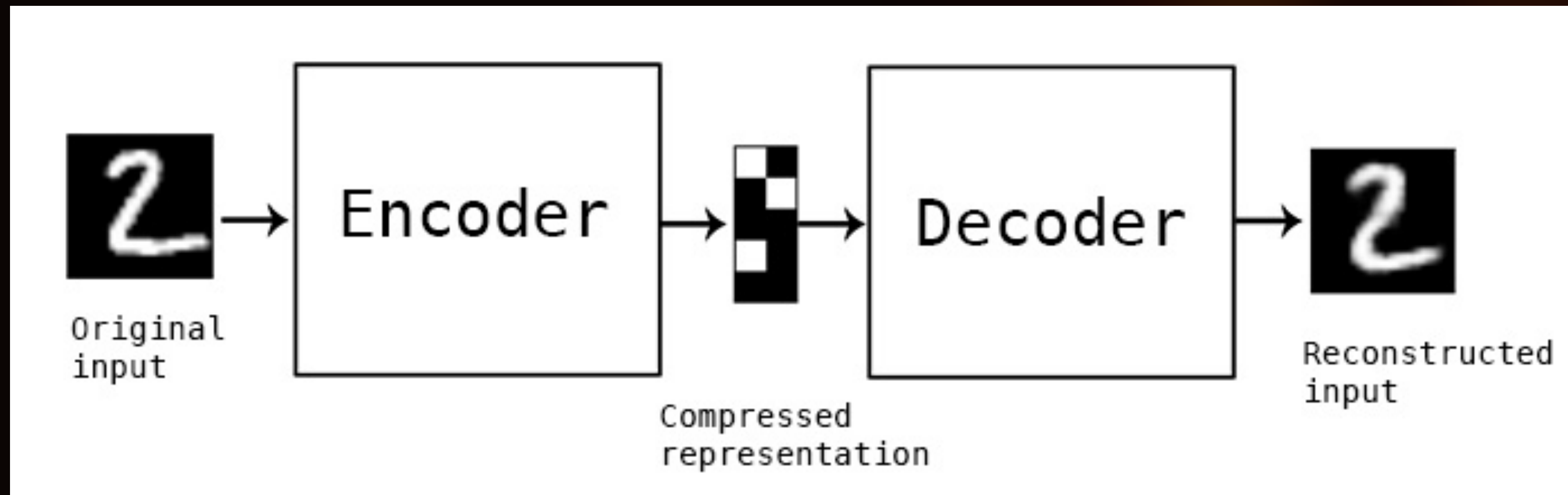
Rosvall & Bergstrom

Bollen et al., 2009

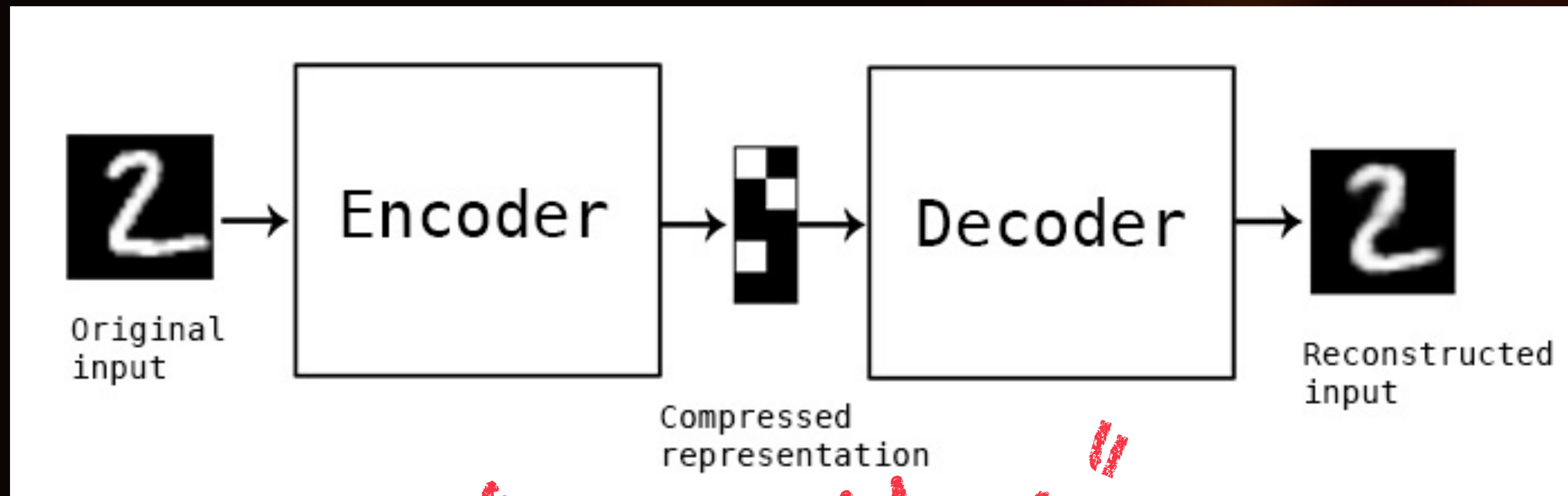


**Can we really push the analogy of
knowledge "space"?**

Deep learning is **representation learning**

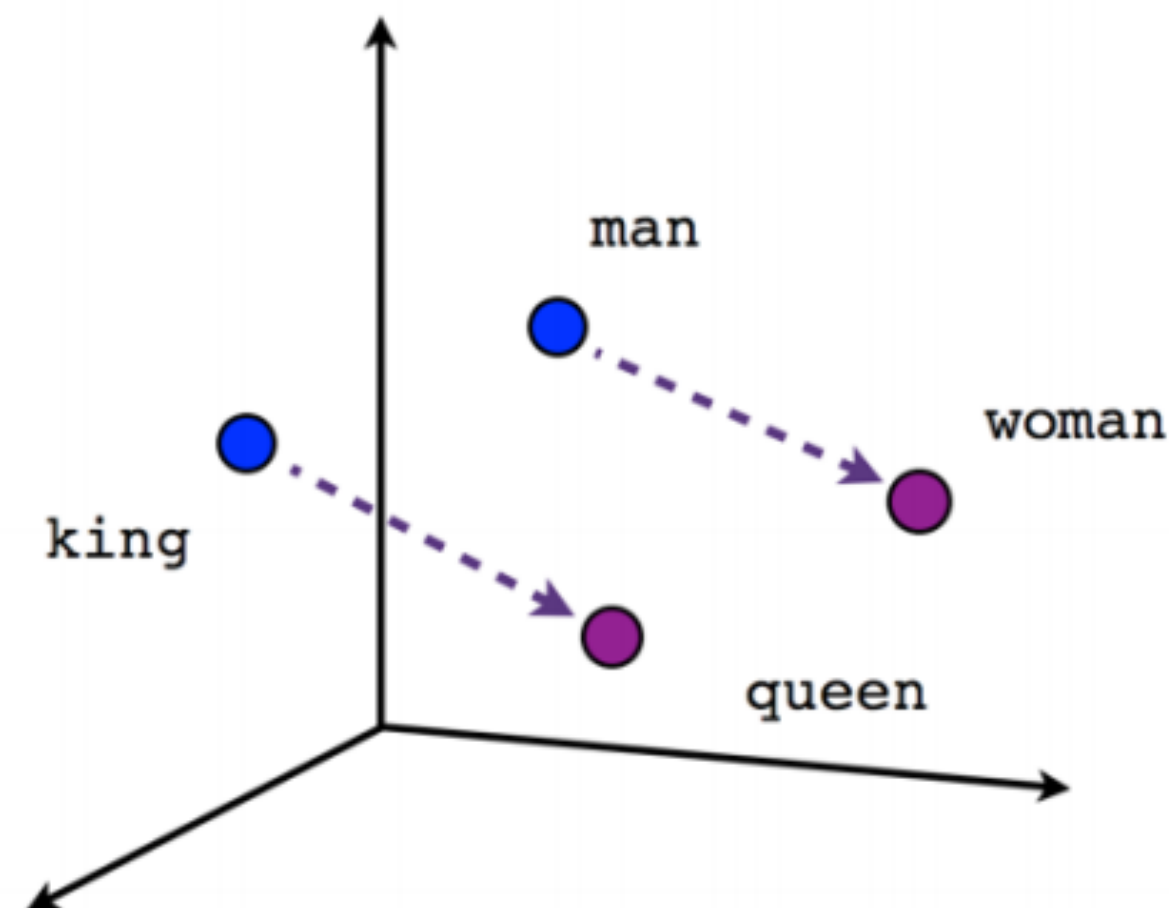


Deep learning is representation learning

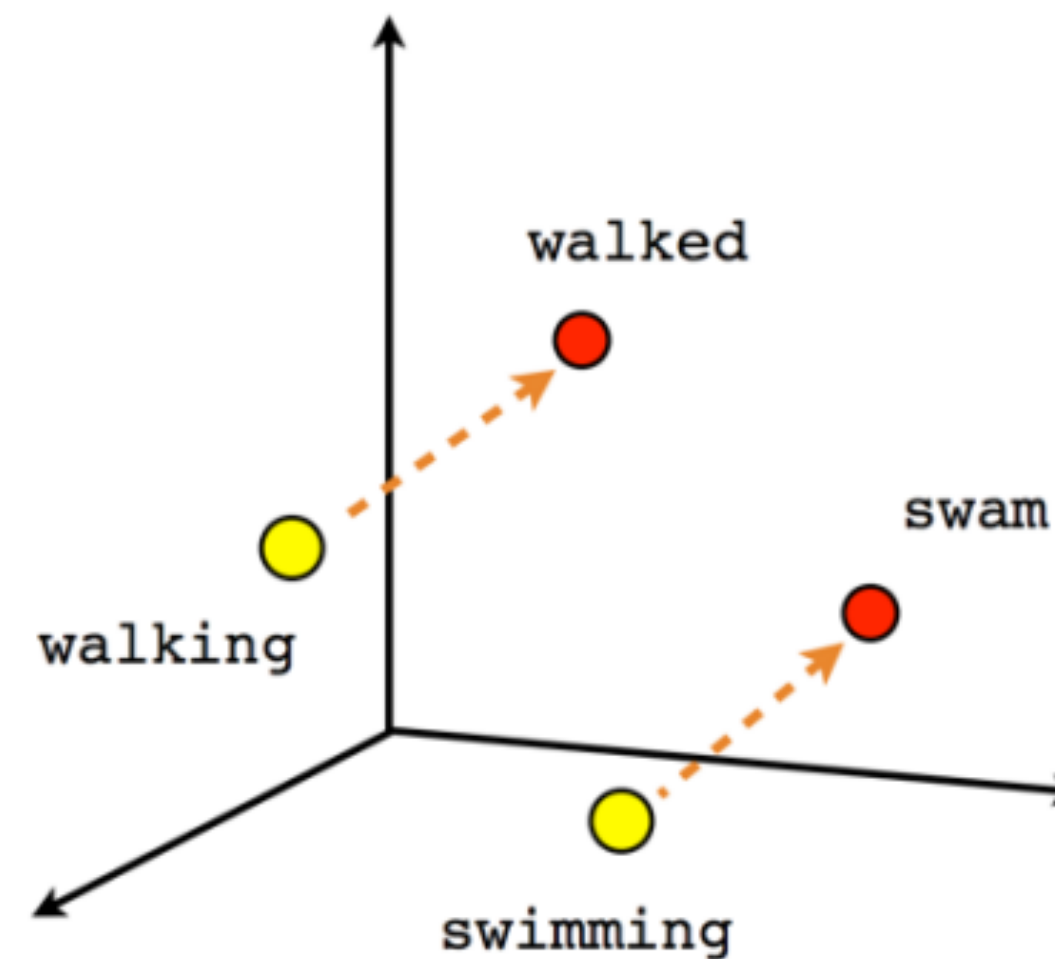


"embedding"

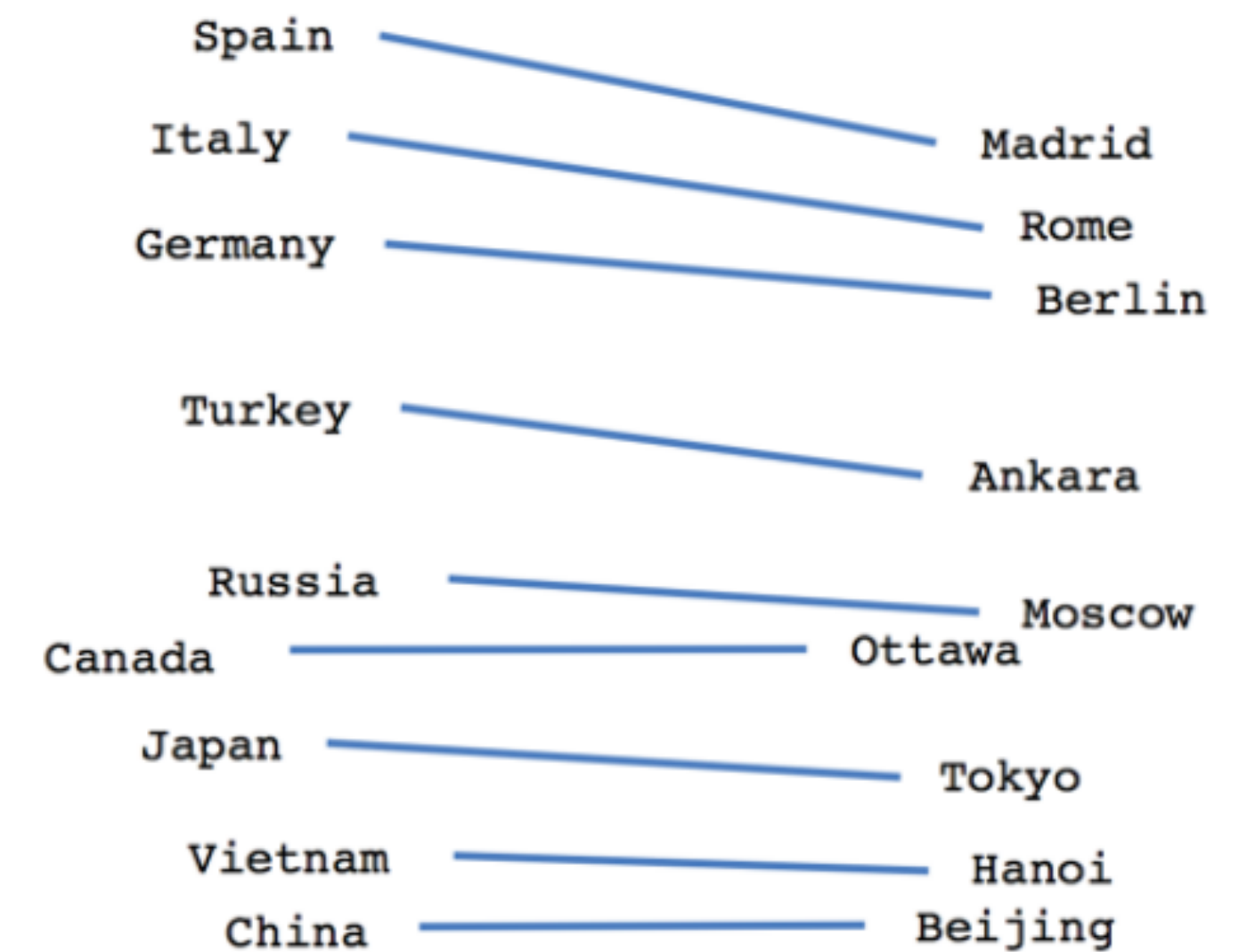
Representation learning can produce interpretable "space"



Male-Female



Verb tense



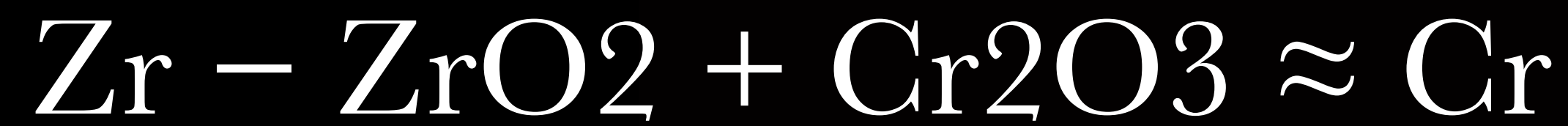
Country-Capital

"Space" of Words,
"Space" of Materials,
"Space" of Images,
"Space" of Knowledge,
...

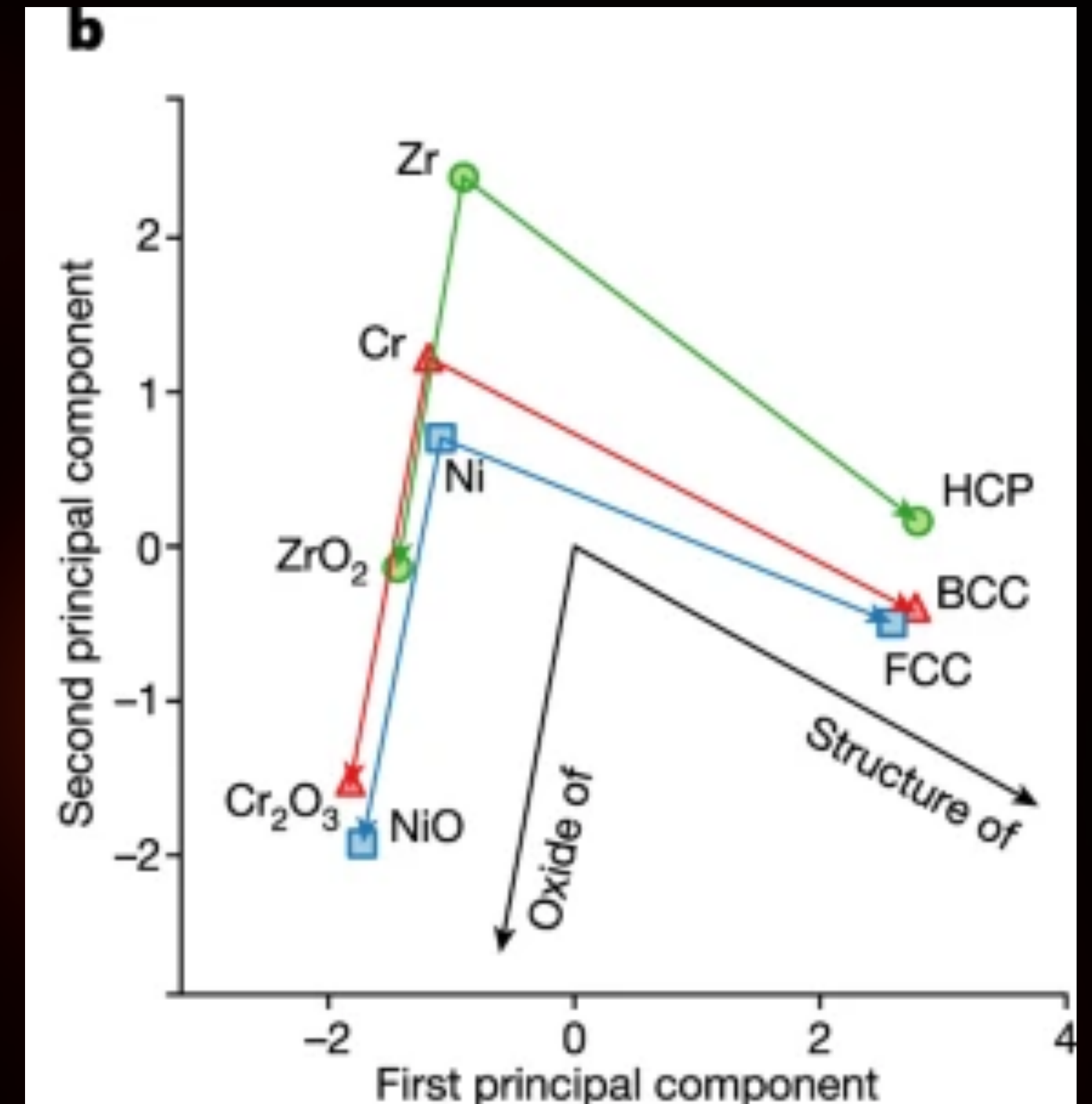
Word embedding → how do words move?

A natural way to trace temporal changes



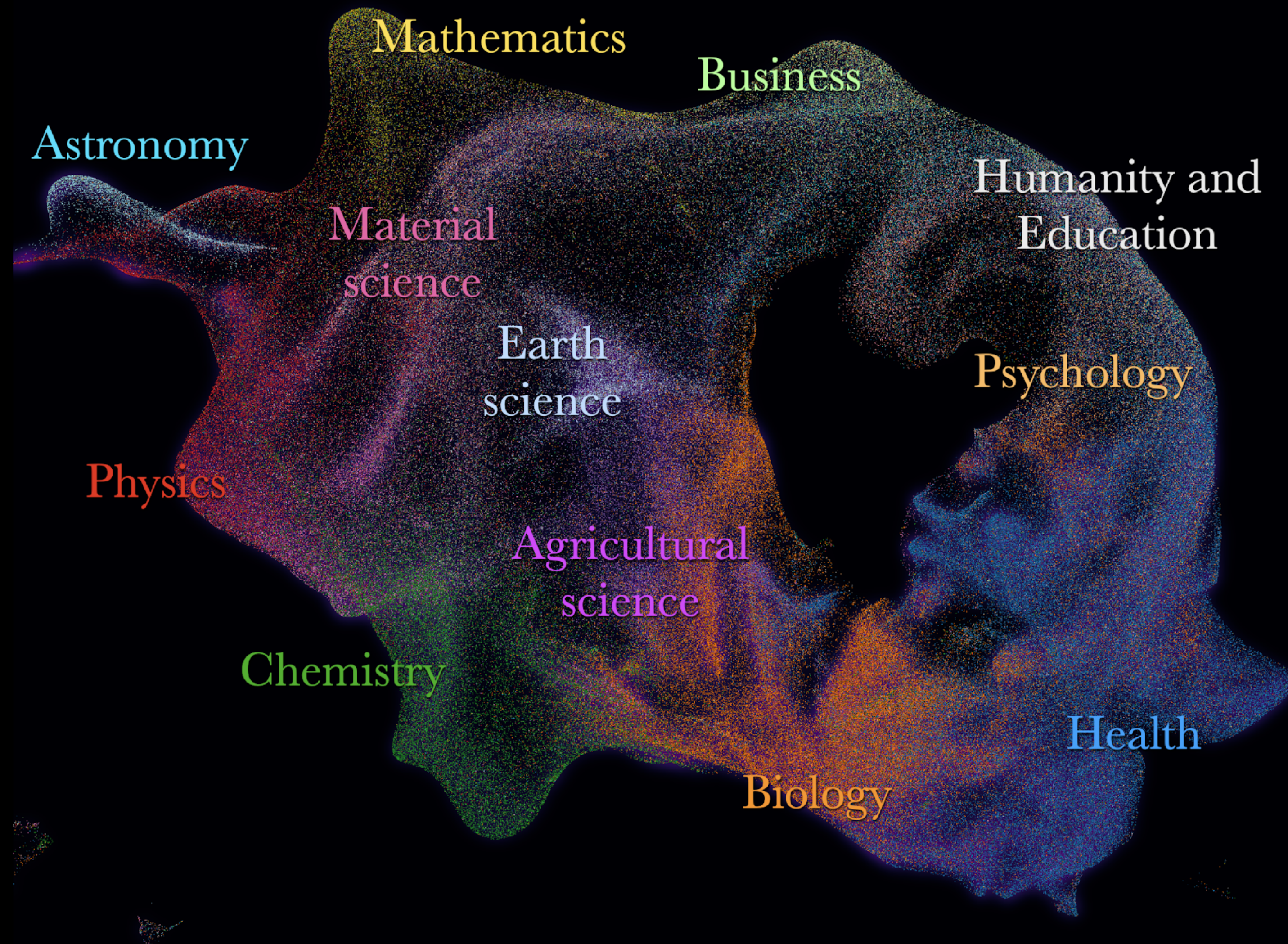


$\overrightarrow{\text{ferromagnetic}} - \overrightarrow{\text{NiFe}} + \overrightarrow{\text{IrMn}}$
 $\approx \overrightarrow{\text{antiferromagnetic}}$



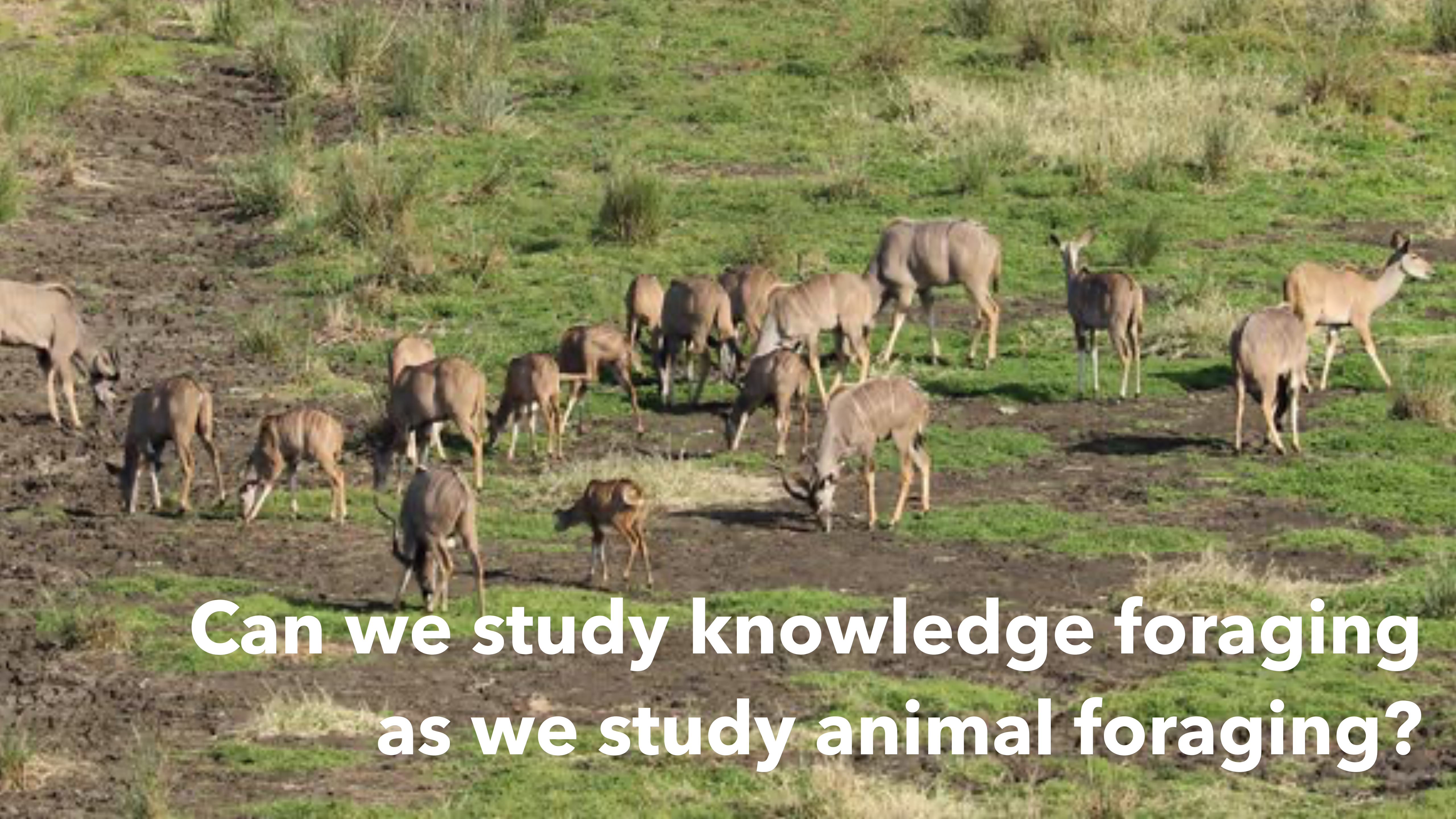
Tshitoyan, V. et al. Nature 571, 95–98 (2019).

Can we imagine the “space of knowledge” as a real “space”?



**Can we model the scientific enterprise
like we model a murmuration of birds?**



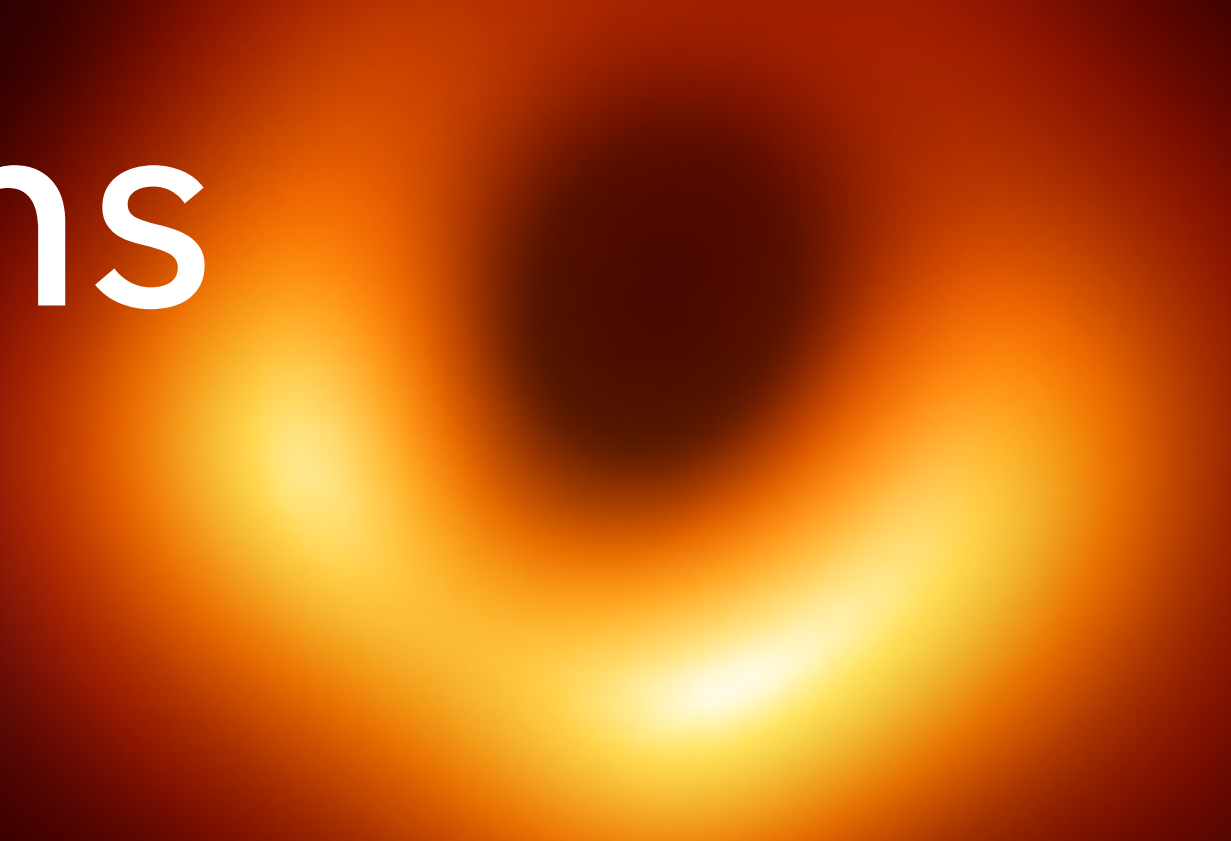


**Can we study knowledge foraging
as we study animal foraging?**



**Can we *imagine and observe* a “blackhole”
in the space of knowledge?**

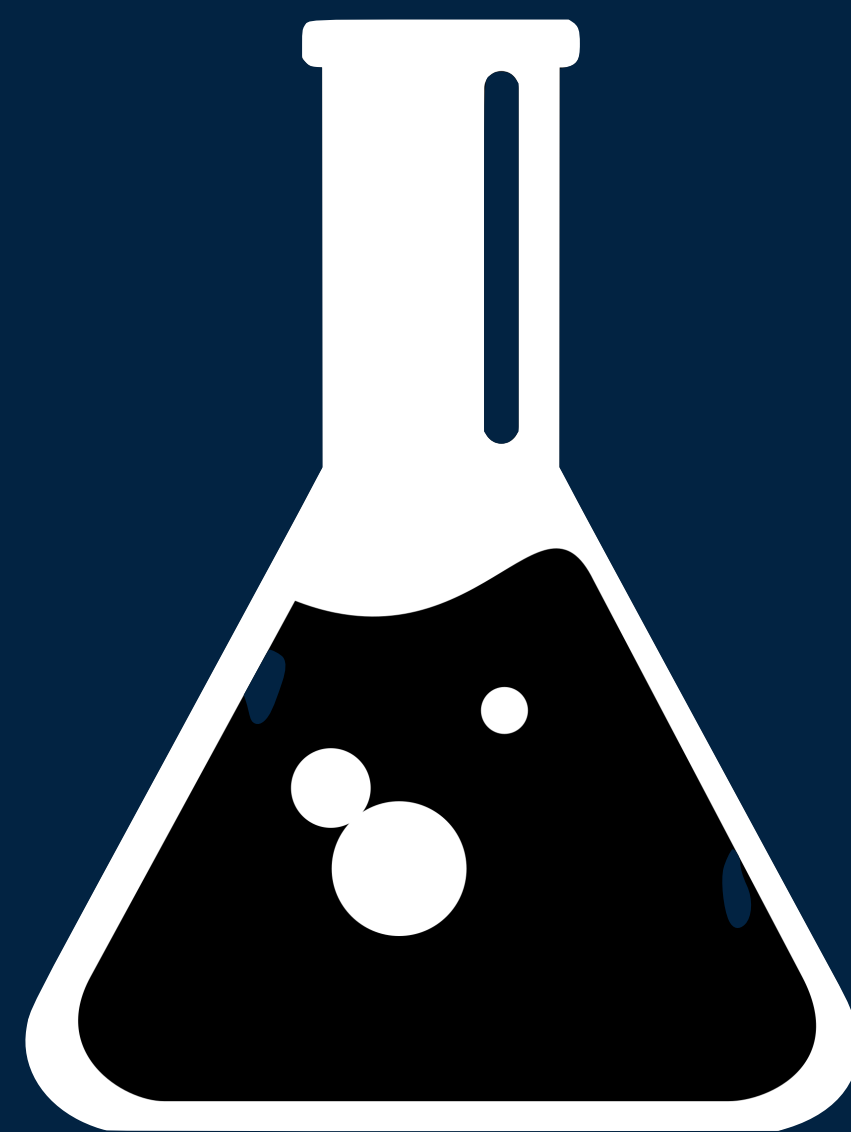
Imagining knowledge space to
model universal citation
dynamics across systems



Law



Science



Patents



	Law (US)	Science	Patents
Participation criteria	Law school, Appointment by president	Anyone in principle, mostly through training	
Choice of subject	Random	Largely driven by personal interests, chosen by participants	
Collaboration	Rare	Common	
Reference discovery	Search engine	Search engine	Search engine
Needs for securing funding	Low	High	?
Publication process	Unilateral	Peer review	USPTO review
Hierarchy	Binding and strict	Not strict	Not strict
Author growth	Slow	High	High
Incentive for citation			Some perverse (?) incentive not to cite the most relevant patents

Law (US) vs. Science

Law (US) vs. Science

- Codified & bounding hierarchy **vs.** organic hierarchy

Law (US) vs. Science

- Codified & bounding hierarchy **vs.** organic hierarchy
- Randomly assigned cases **vs.** personal choice of problems.

Law (US) vs. Science

- Codified & bounding hierarchy **vs.** organic hierarchy
- Randomly assigned cases **vs.** personal choice of problems.
- Much fewer, rather fixed number of judges (~6,000 judges) **vs.** exponentially growing pool of scientists

Law (US) vs. Science

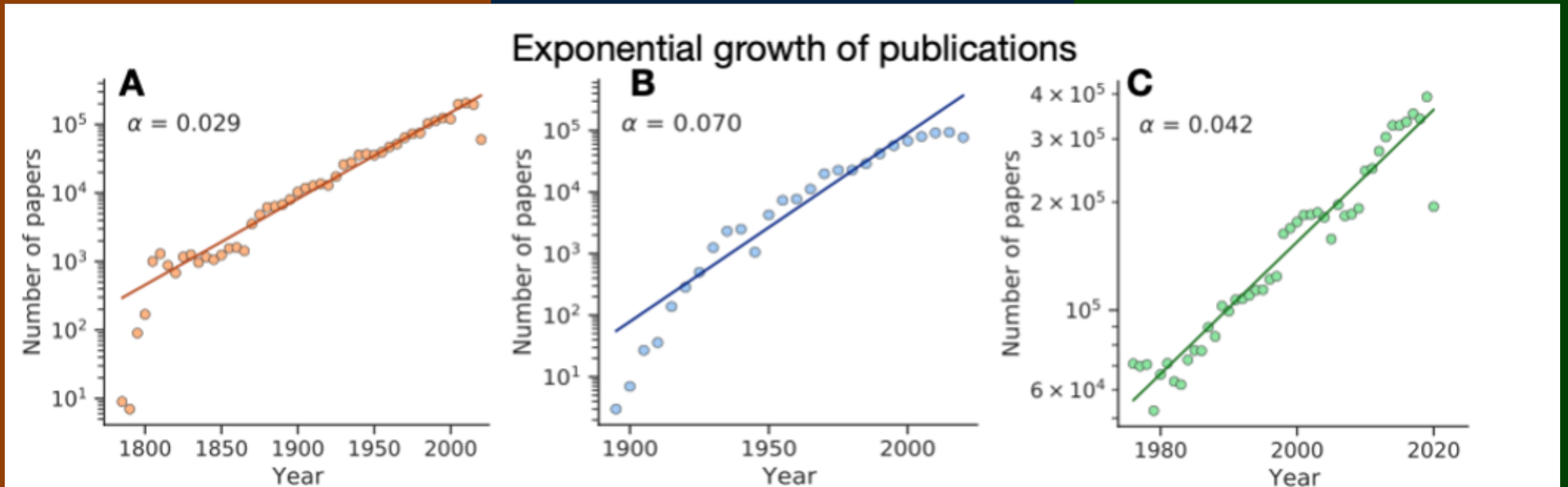
- Codified & bounding hierarchy **vs.** organic hierarchy
- Randomly assigned cases **vs.** personal choice of problems.
- Much fewer, rather fixed number of judges (~6,000 judges) **vs.** exponentially growing pool of scientists
- Life-term appointments and no direct motivation to get citations **vs.** publish (and get cited) or perish

Law (US) vs. Science

- Codified & bounding hierarchy **vs.** organic hierarchy
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- Life-term appointments and no direct motivation to get citations **vs.** publish (and get cited) or perish
- ...

Different modes of
operation, yet

Exponential growth



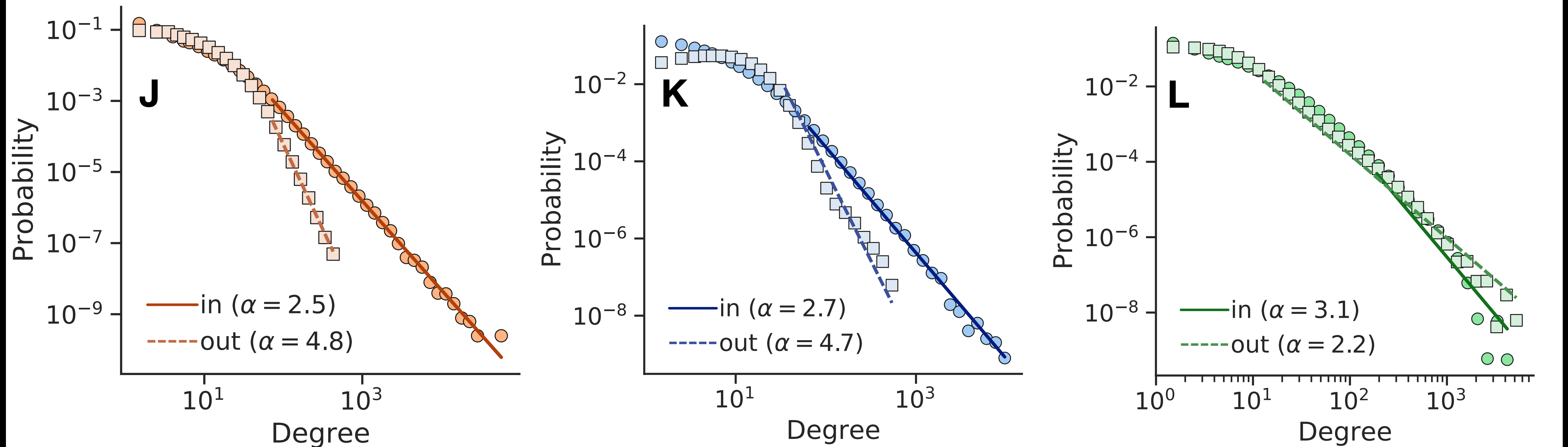
Law

Science

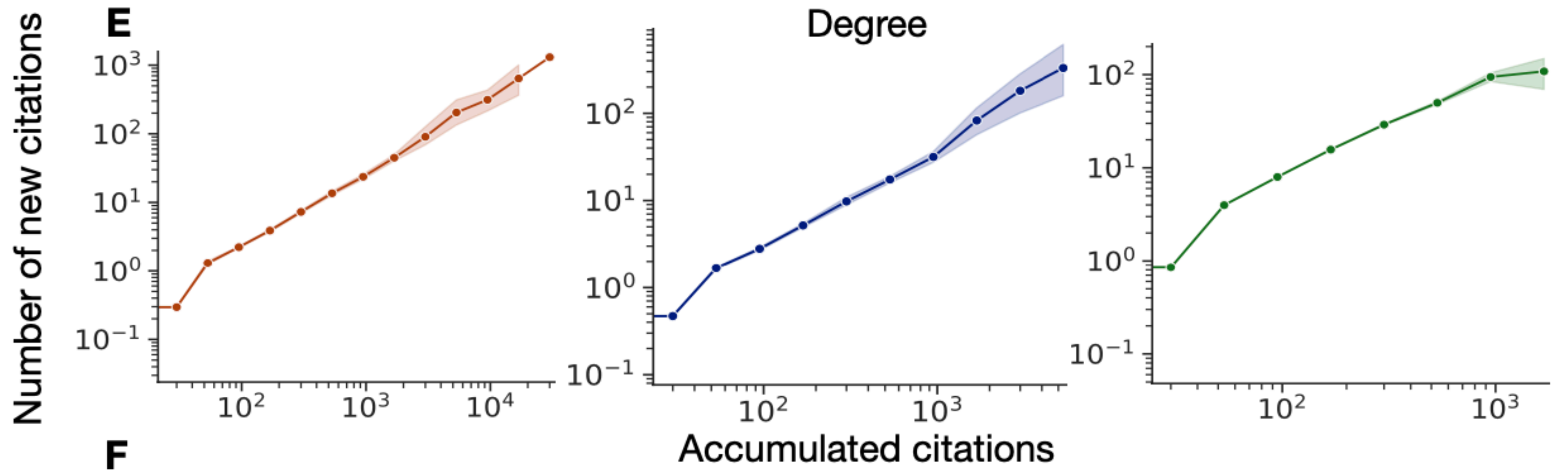
Patents

Degree distribution

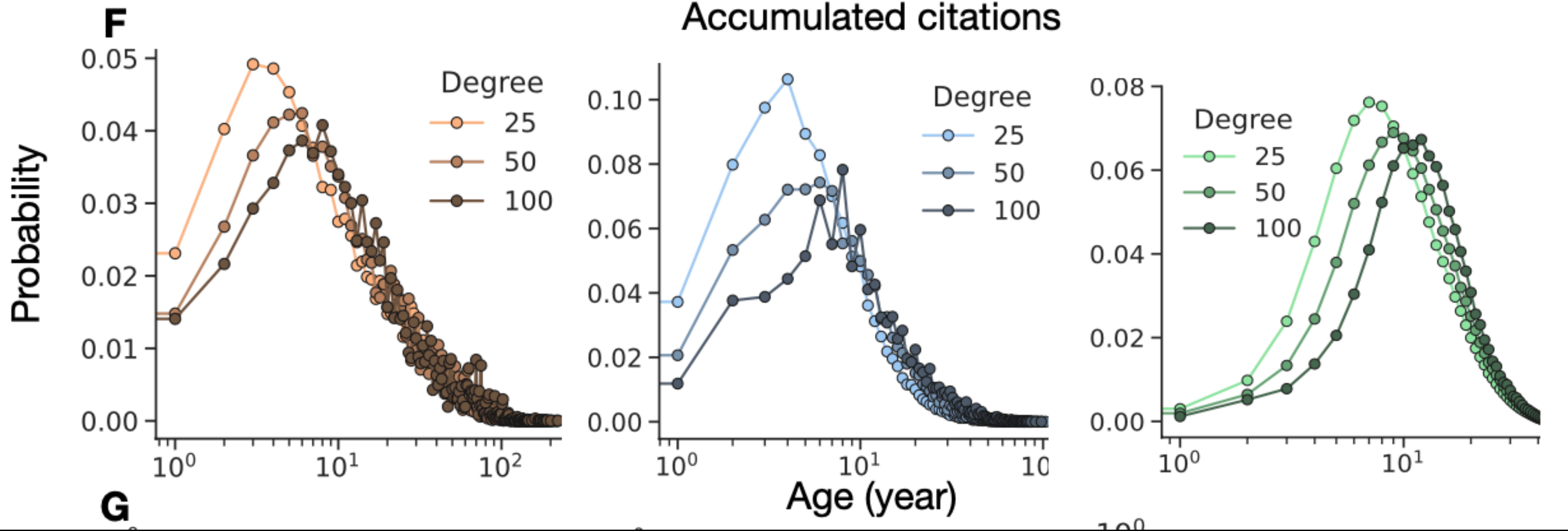
Heterogeneous degree distribution



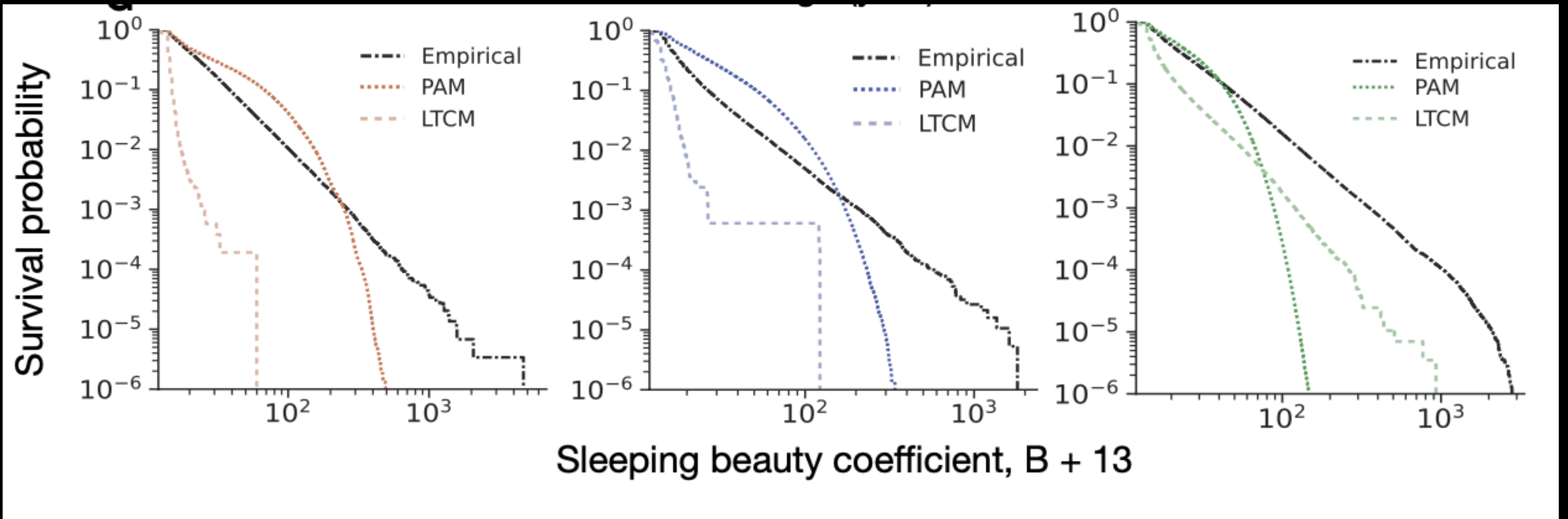
Preferential Attachment



Citation recency



Delayed recognition



Then, what is the
underlying mechanism?

$$\Pi_i(t) \sim \eta_i c_i^t P_i(t)$$

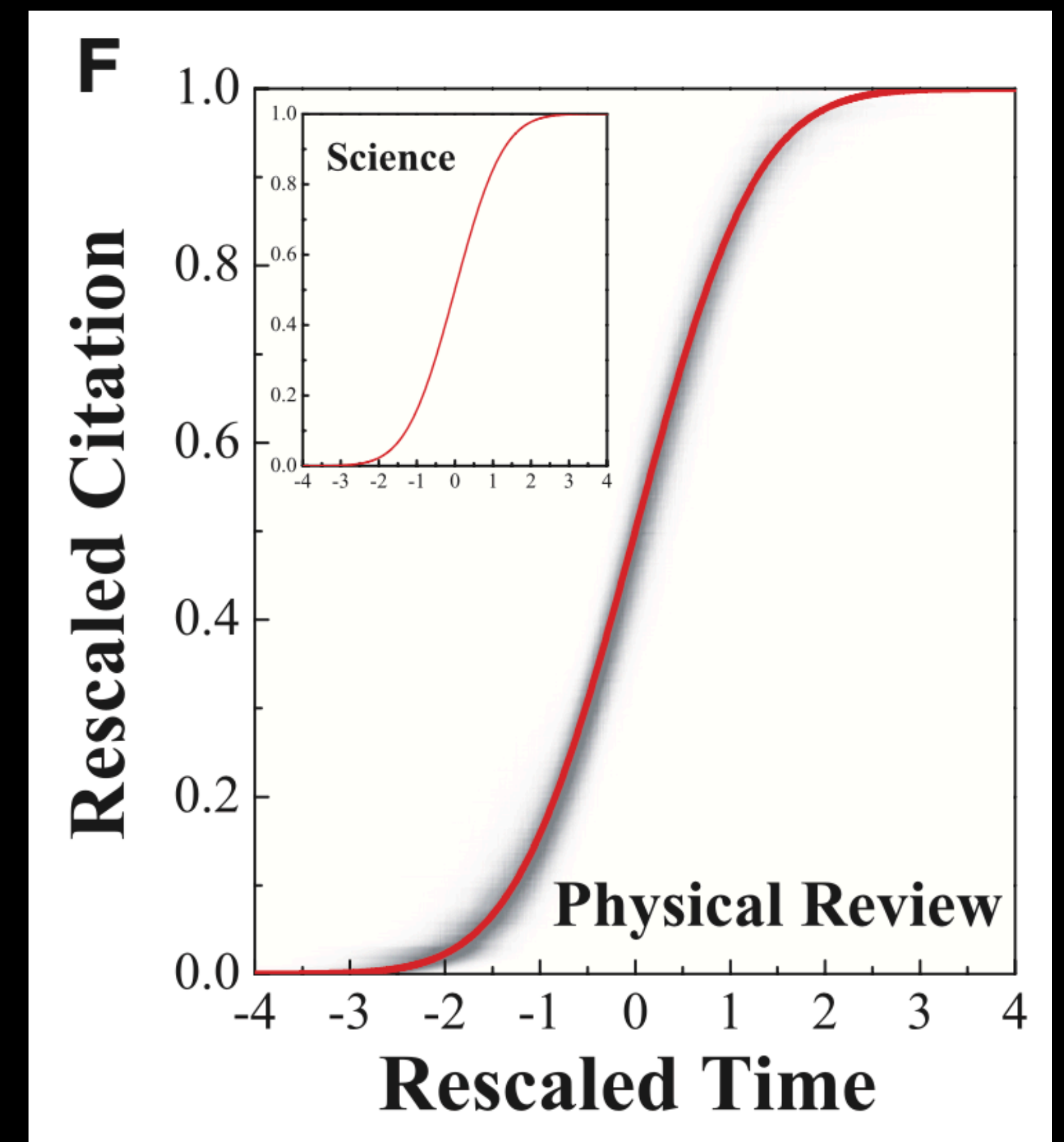
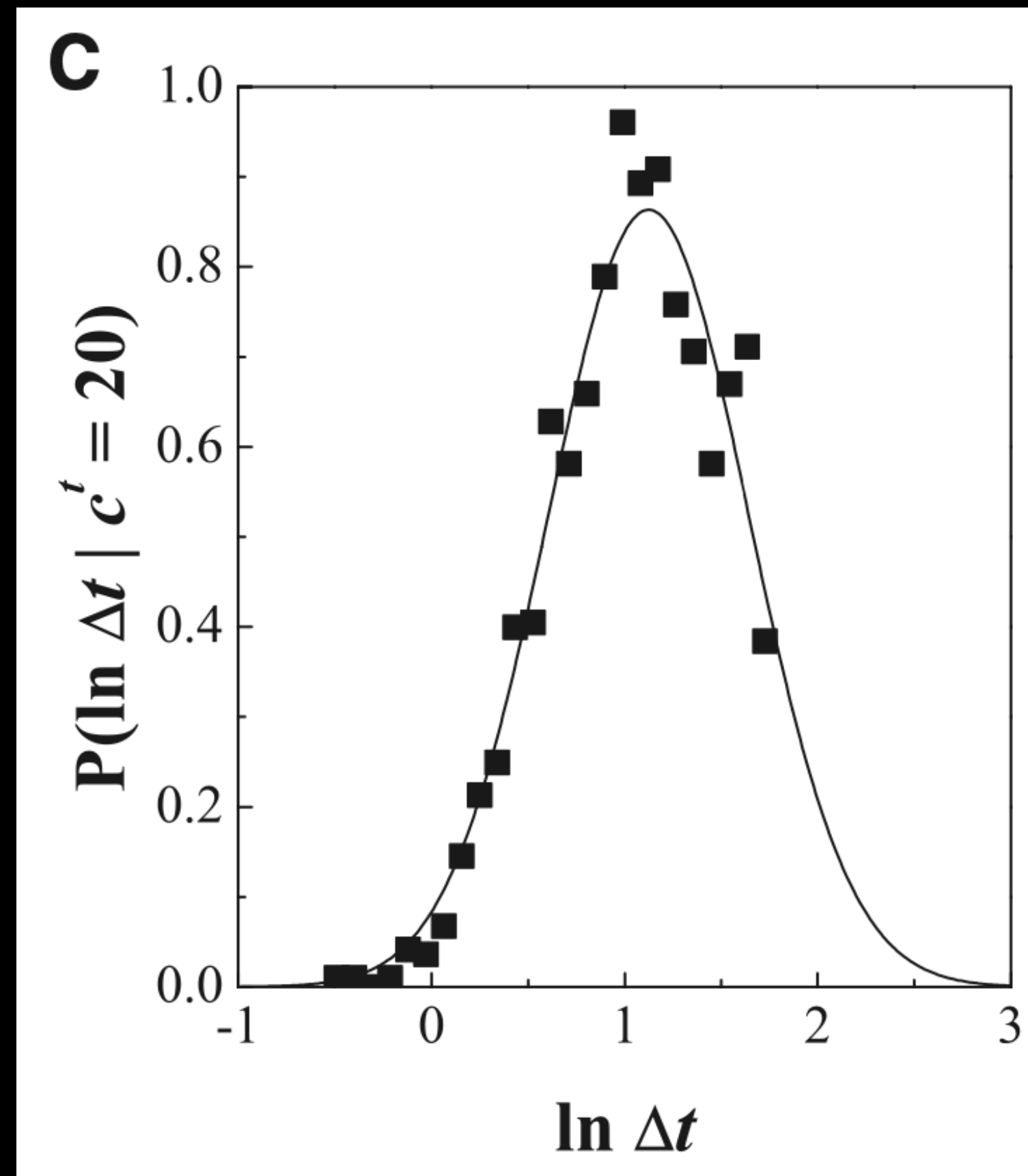
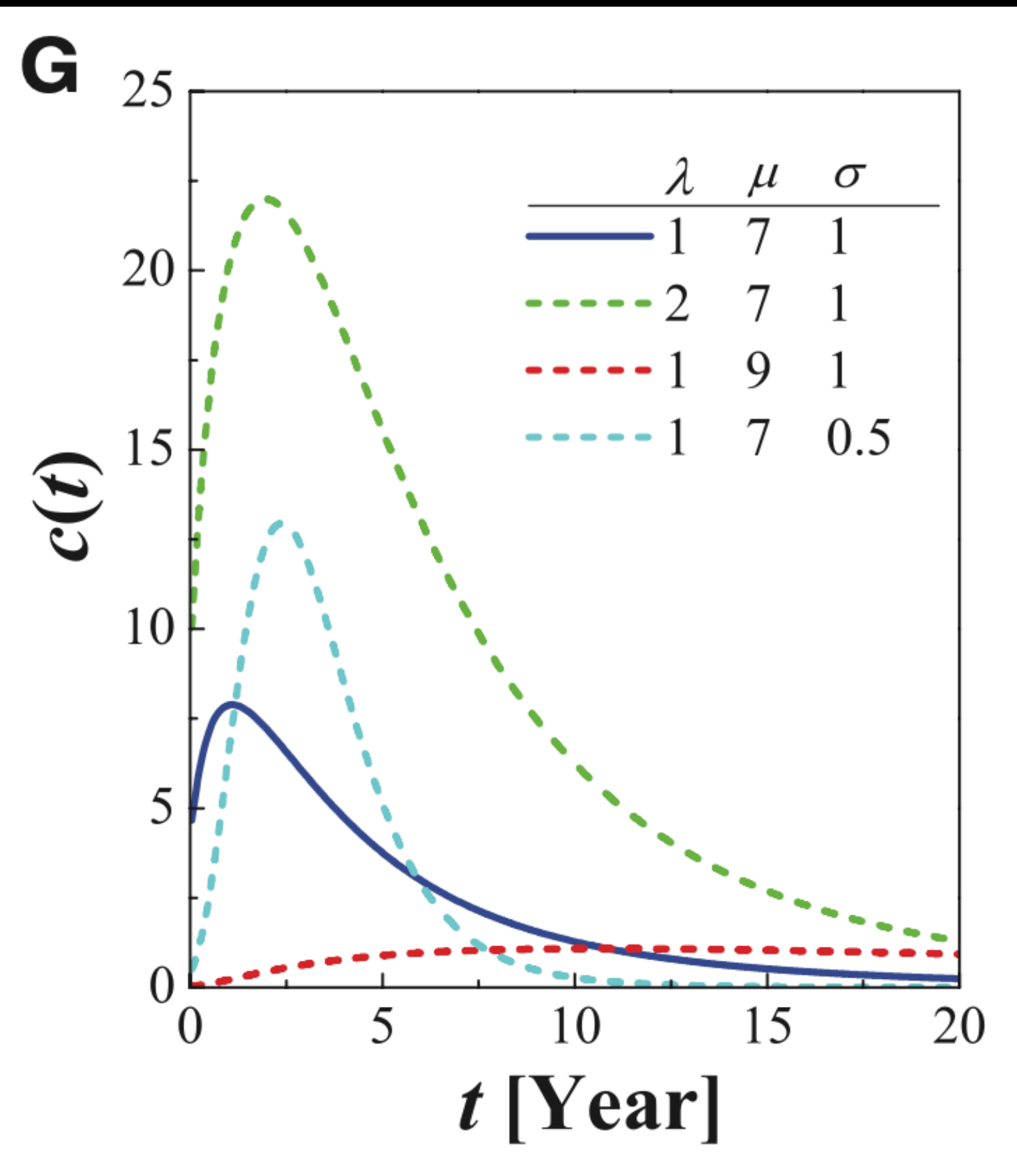
$$\Pi_i(t) \sim \underline{\eta_i} c_i^t P_i(t)$$

Fitness

$$\Pi_i(t) \sim \underbrace{\eta_i}_{\text{Fitness}} \overbrace{c_i^t}^{\text{Citation (PA)}} P_i(t)$$

$$\Pi_i(t) \sim \underbrace{\eta_i}_{\text{Fitness}} \underbrace{c_i^t}_{\text{Citation (PA)}} \underbrace{P_i(t)}_{\text{Aging}}$$

Long-term citation model



But,

*“Performance is about **you**. . . .
Success, however, is about **us**.”*

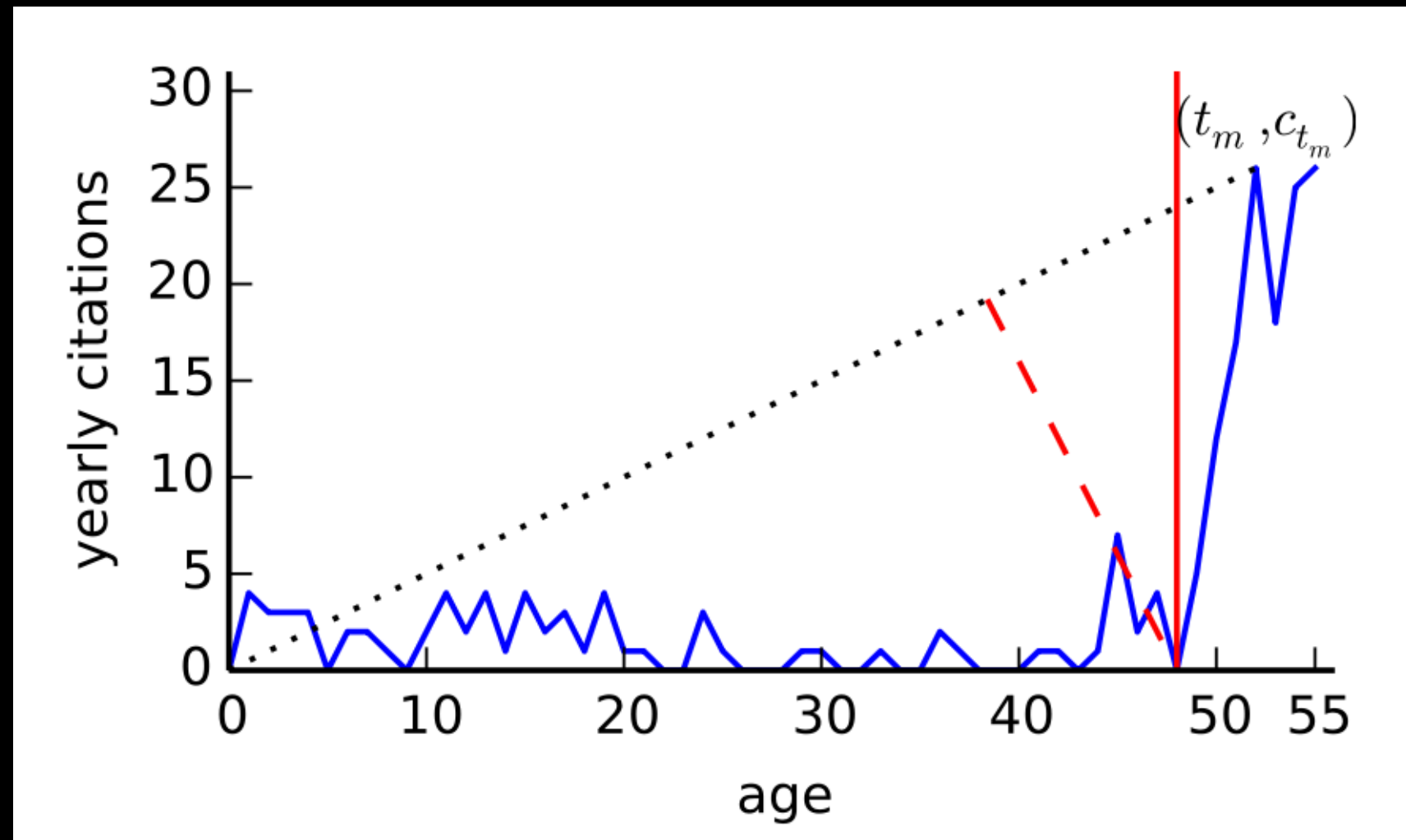
—Albert-László Barabási

Citation $\sim$ success?

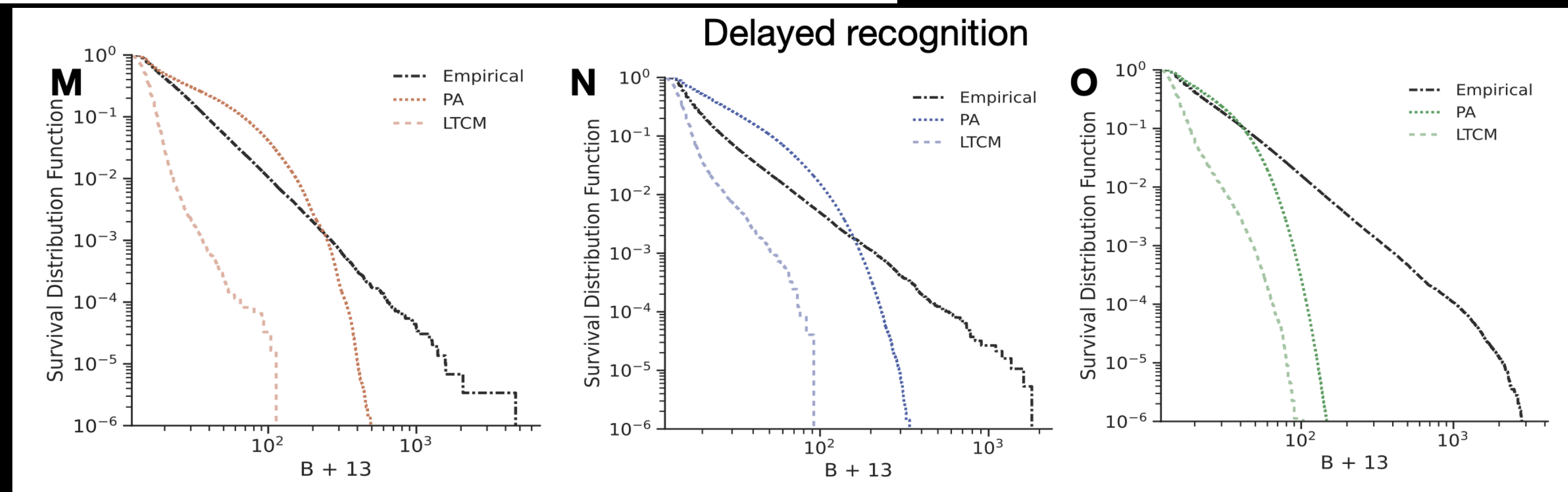
Then, is it really enough to only focus on the *paper* (**you**), not the *collective/community* (**us**)?

Shouldn't the citation dynamics be inherently **collective**?

An example: **delayed recognition** (“sleeping beauties”)



$$\Pi_i(t) \sim ? c_i^t P_i(t)$$



Community citation model

Community citation model

- **Paper-centric view**: the success is determined by its *intrinsic quality, age, and luck*.

Community citation model

- **Paper-centric view**: the success is determined by its *intrinsic quality, age, and luck*.
- **Collective / community-centric view**: the success (citations) is determined by the *collective attention of the community*.

How can we model the
“*community*”?

The space of knowledge



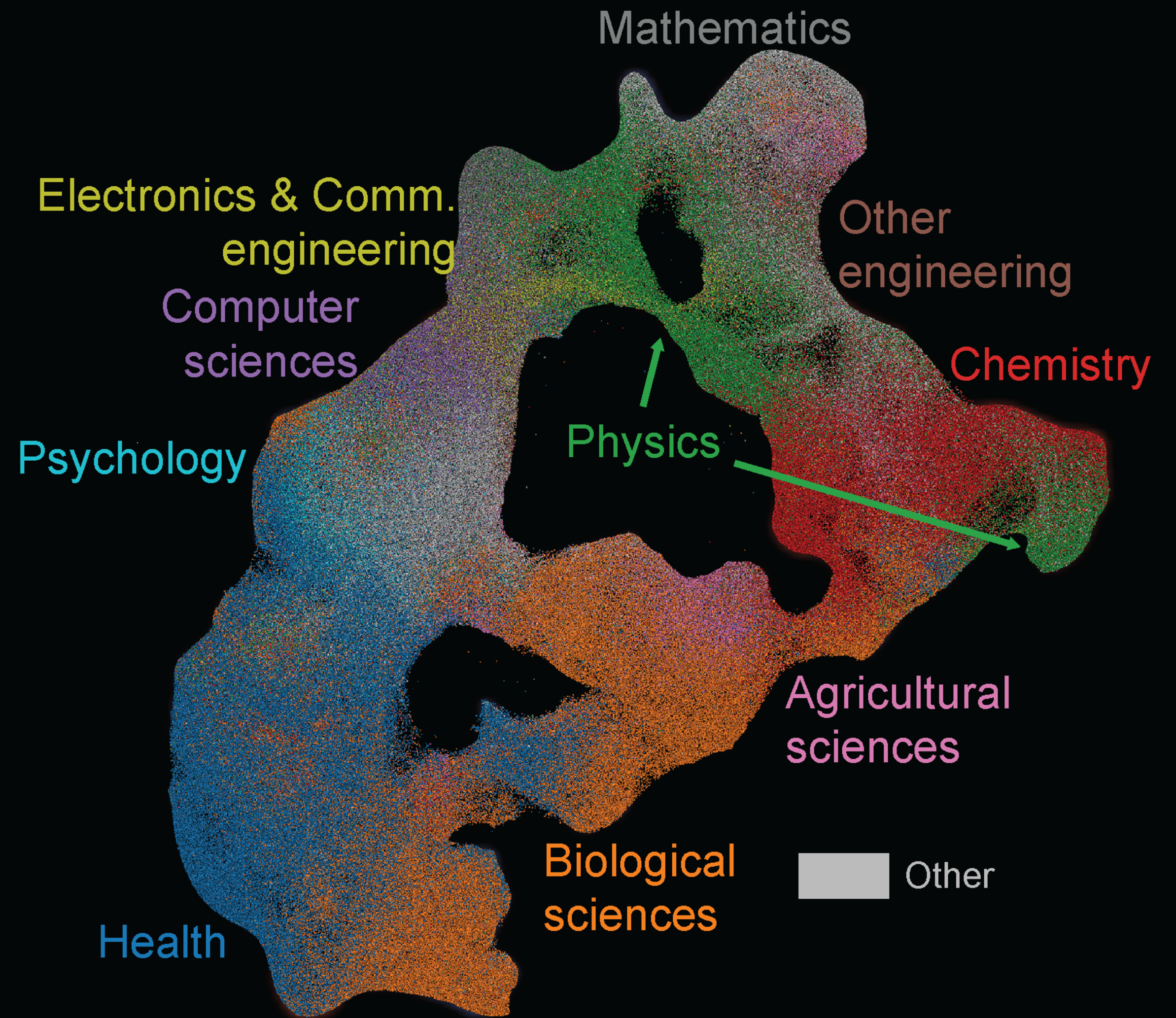
Filipi Silva



Sadamori Kojaku



Lili Miao



Existing papers

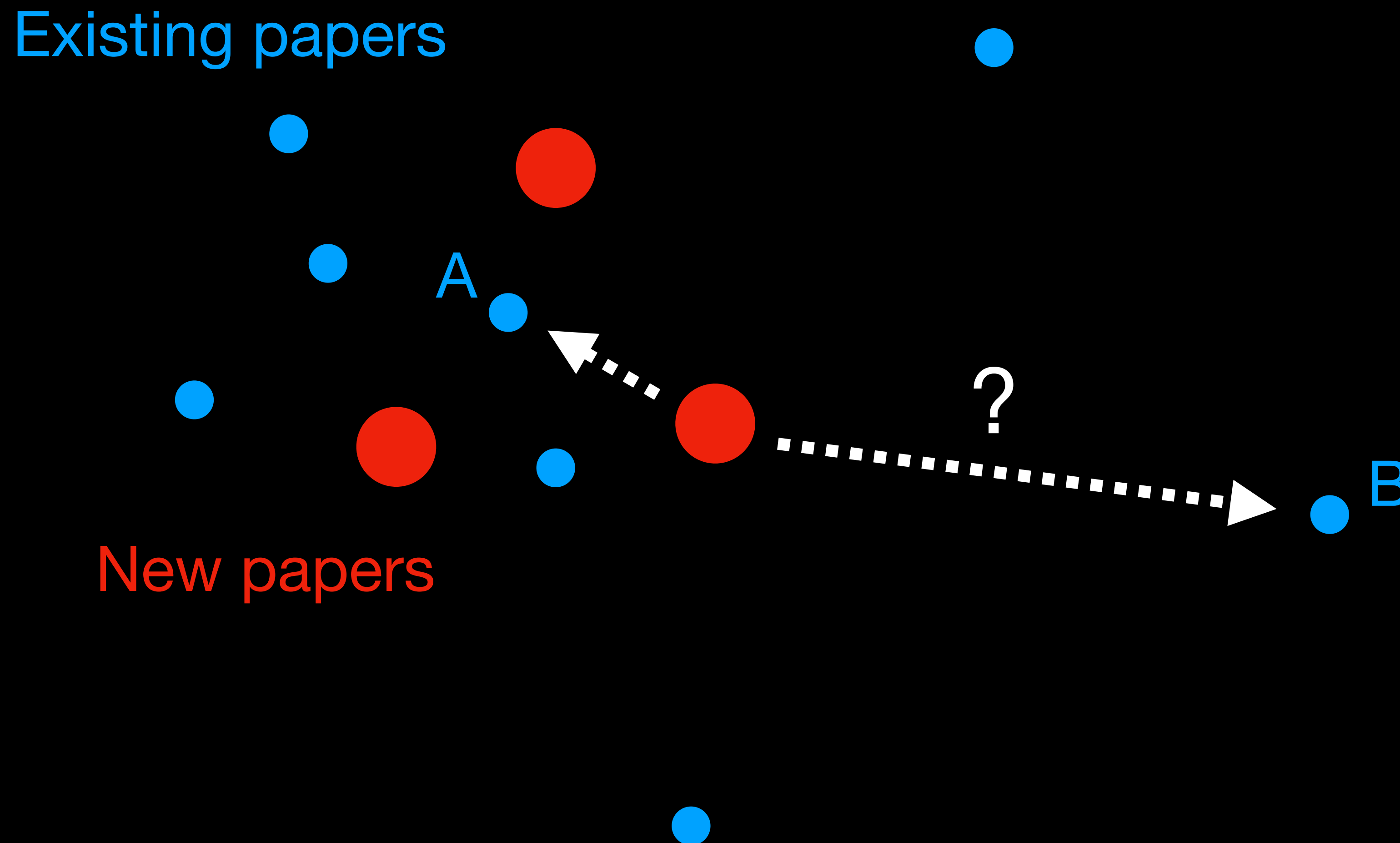
New papers

A

?

B

The space of knowledge

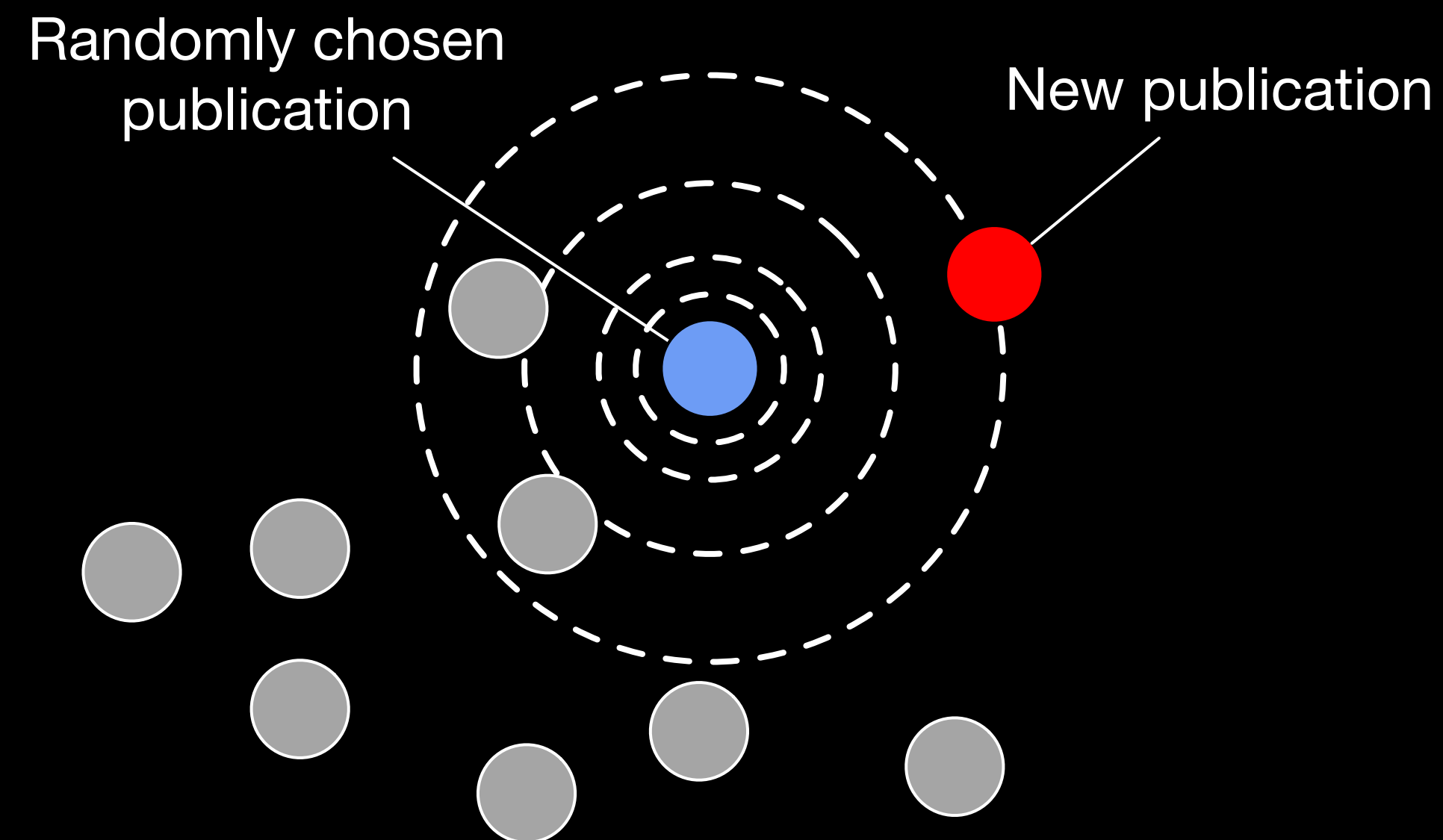


Community citation model

1. New knowledge (papers) *get created somewhere* in the space of knowledge
2. Each paper *identifies relevant existing literature* and cite them.

Adjacent production model

A existing paper is randomly chosen, and then a new paper appears near the chosen paper:



$$P(u_i | u_j) \propto \exp(-\lambda d(\mathbf{u}_i, \mathbf{u}_j))$$

von Mises–Fisher distribution

How do they identify relevant knowledge?

How do they identify relevant knowledge?

$$\Pi_i(t) \sim \underbrace{\eta_i}_{\text{Fitness}} \underbrace{c_i^t}_{\text{Citation (PA)}} \underbrace{P_i(t)}_{\text{Aging}}$$

Community citation model

$$P(j|i) = \frac{1}{Z_{t_i}} \eta_j \left(c_j(t_i) + c_0 \right) f(t_i - t_j) \cdot \exp \left(-\kappa d(\mathbf{u}_j, \mathbf{u}_i) \right)$$

Community citation model

$$P(j|i) = \frac{1}{Z_{t_i}} \eta_j \left(c_j(t_i) + c_0 \right) f(t_i - t_j) \cdot \exp \left(-\kappa d(\mathbf{u}_j, \mathbf{u}_i) \right)$$

Fitness

Community citation model

$$P(j|i) = \frac{1}{\underbrace{Z_{t_i}}_{\text{Fitness}}} \overbrace{\eta_j (c_j(t_i) + c_0)}^{\text{Citation (PA)}} f(t_i - t_j) \cdot \exp(-\kappa d(\mathbf{u}_j, \mathbf{u}_i))$$

Community citation model

$$P(j|i) = \frac{1}{Z_{t_i}} \eta_j \left(\overbrace{c_j(t_i) + c_0}^{\text{Citation (PA)}} \right) \underbrace{f(t_i - t_j)}_{\text{Aging}^*} \cdot \exp \left(-\kappa d(\mathbf{u}_j, \mathbf{u}_i) \right)$$

Fitness

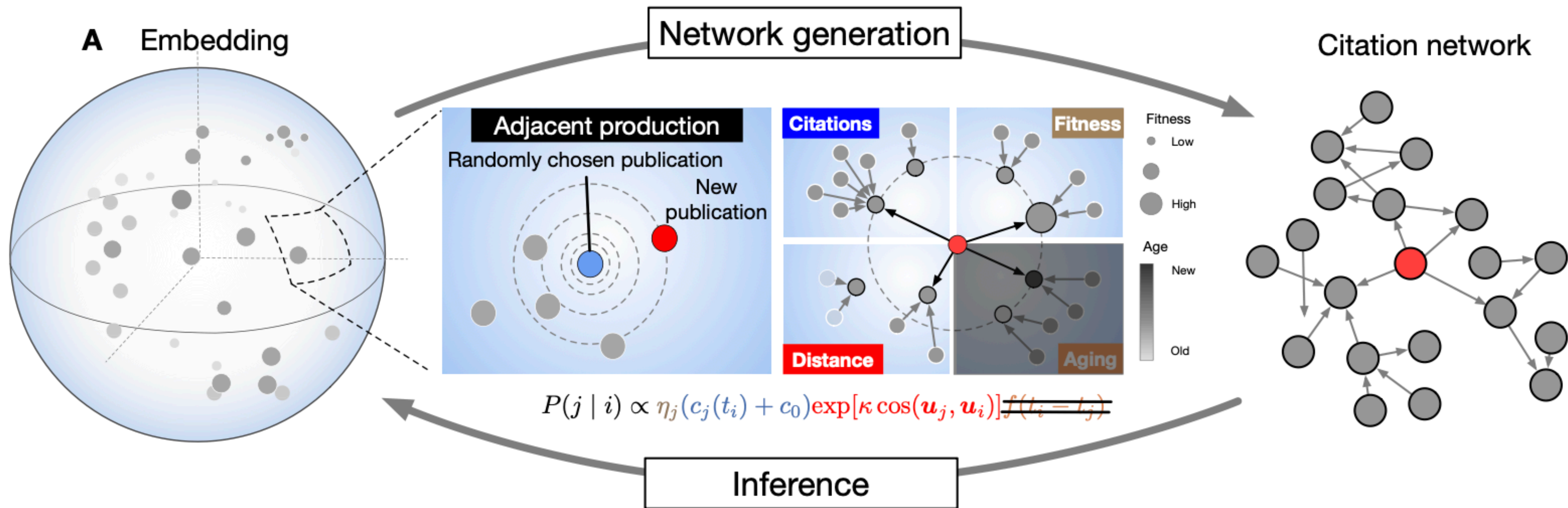
Community citation model

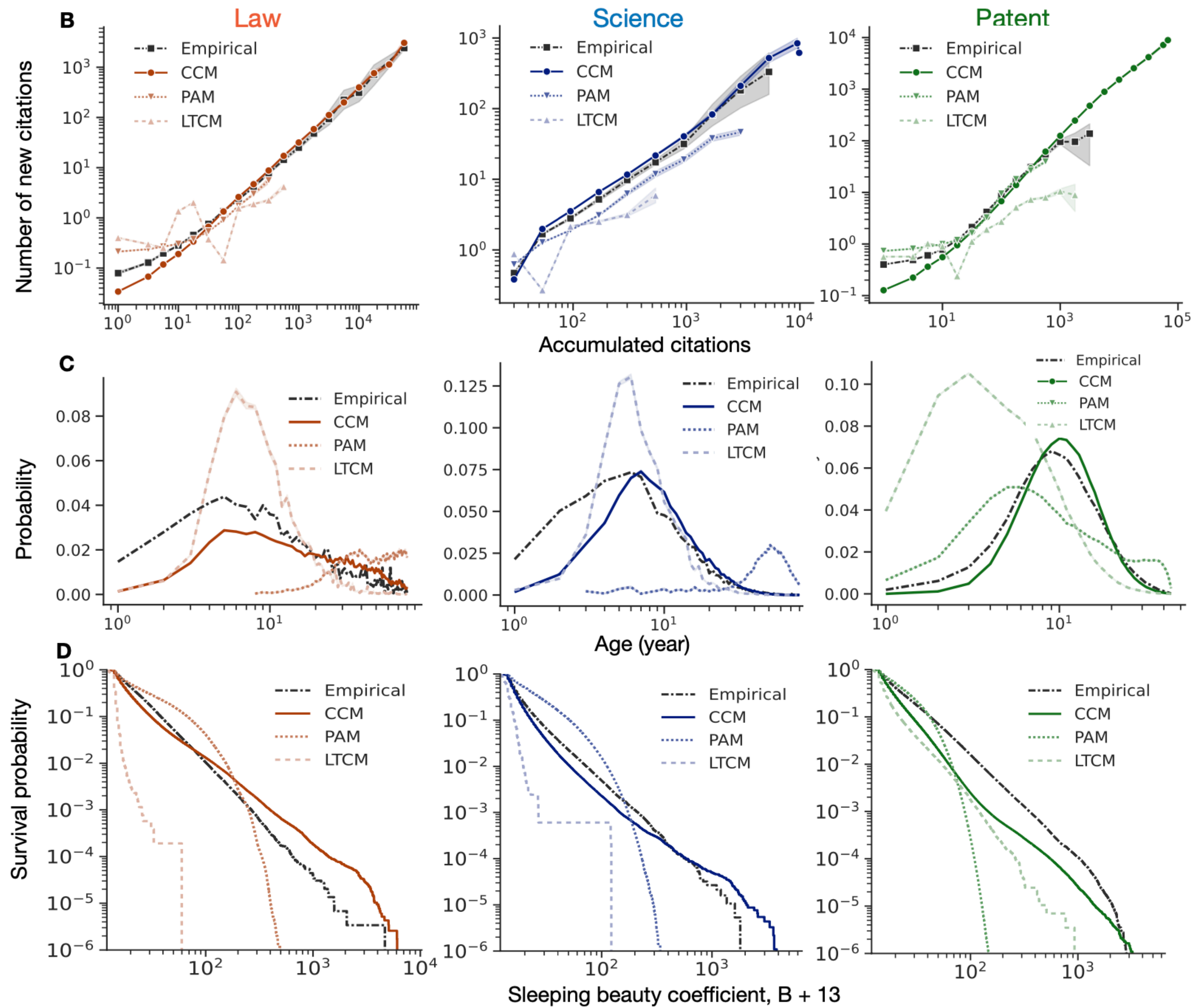
$$P(j|i) = \frac{1}{Z_{t_i}} \eta_j \left(\overbrace{c_j(t_i) + c_0}^{\text{Citation (PA)}} \right) \underbrace{f(t_i - t_j)}_{\text{Aging}^*} \cdot \exp \left(-\kappa d(\underbrace{\mathbf{u}_j, \mathbf{u}_i}_{\text{Embedding vectors}}) \right)$$

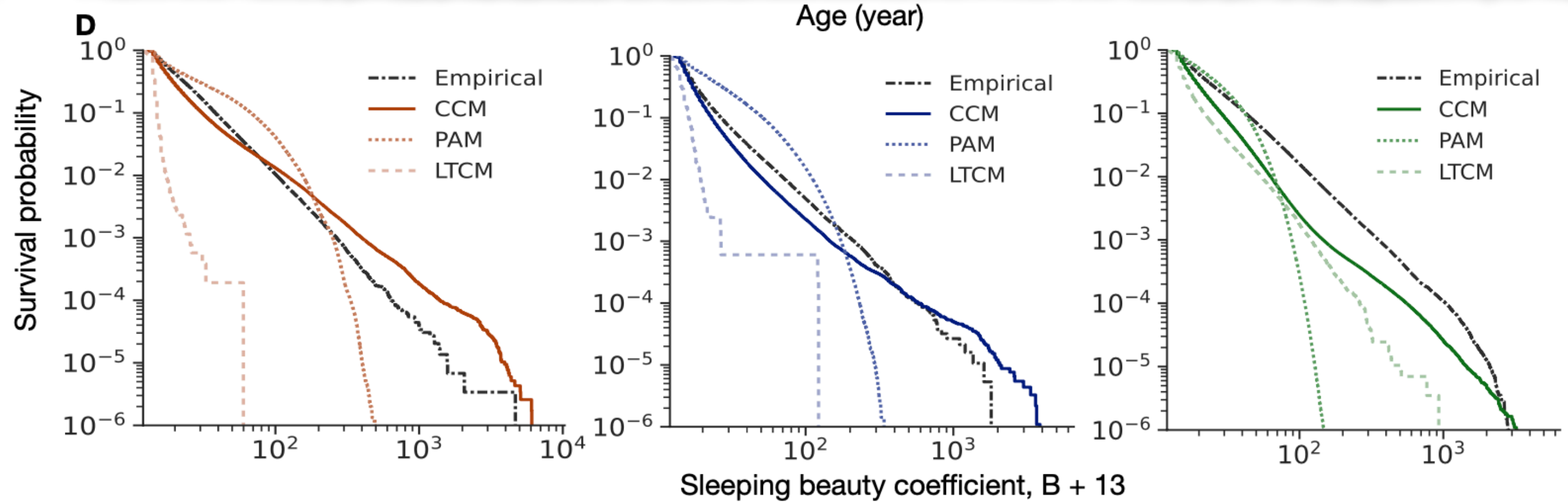
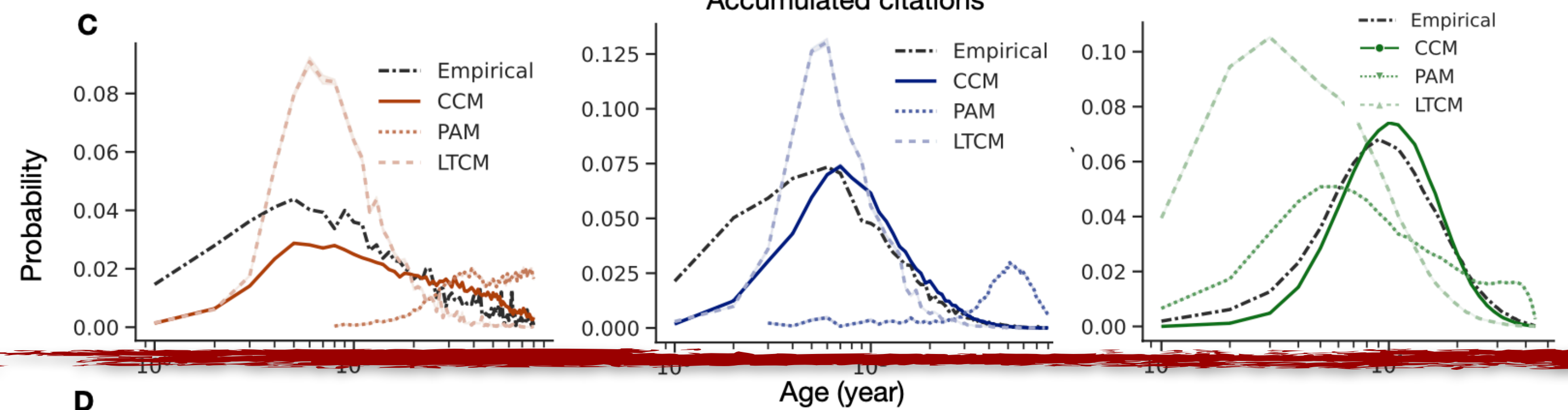
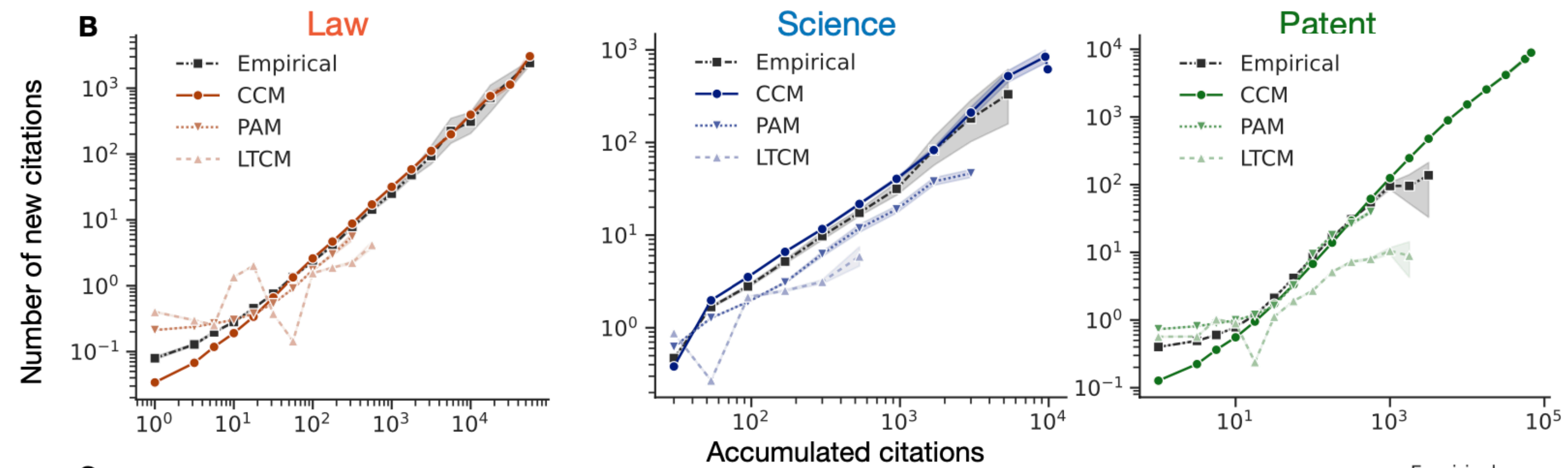
Community citation model

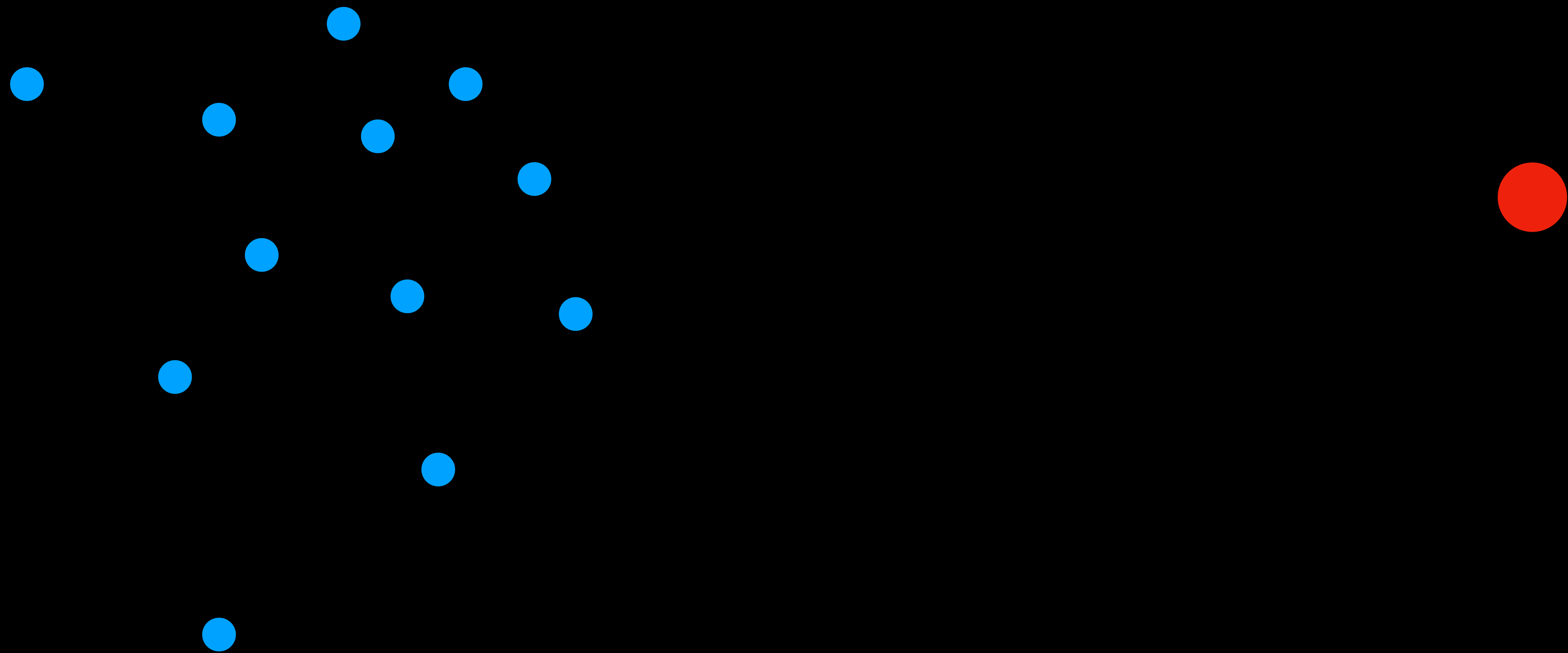
$$P(j|i) = \frac{1}{Z_{t_i}} \eta_j \left(\overbrace{c_j(t_i) + c_0}^{\text{Citation (PA)}} \right) \underbrace{f(t_i - t_j)}_{\text{Aging}^*} \cdot \exp \left(-\kappa d(\underbrace{\mathbf{u}_j, \mathbf{u}_i}_{\text{Embedding vectors}}) \right)$$

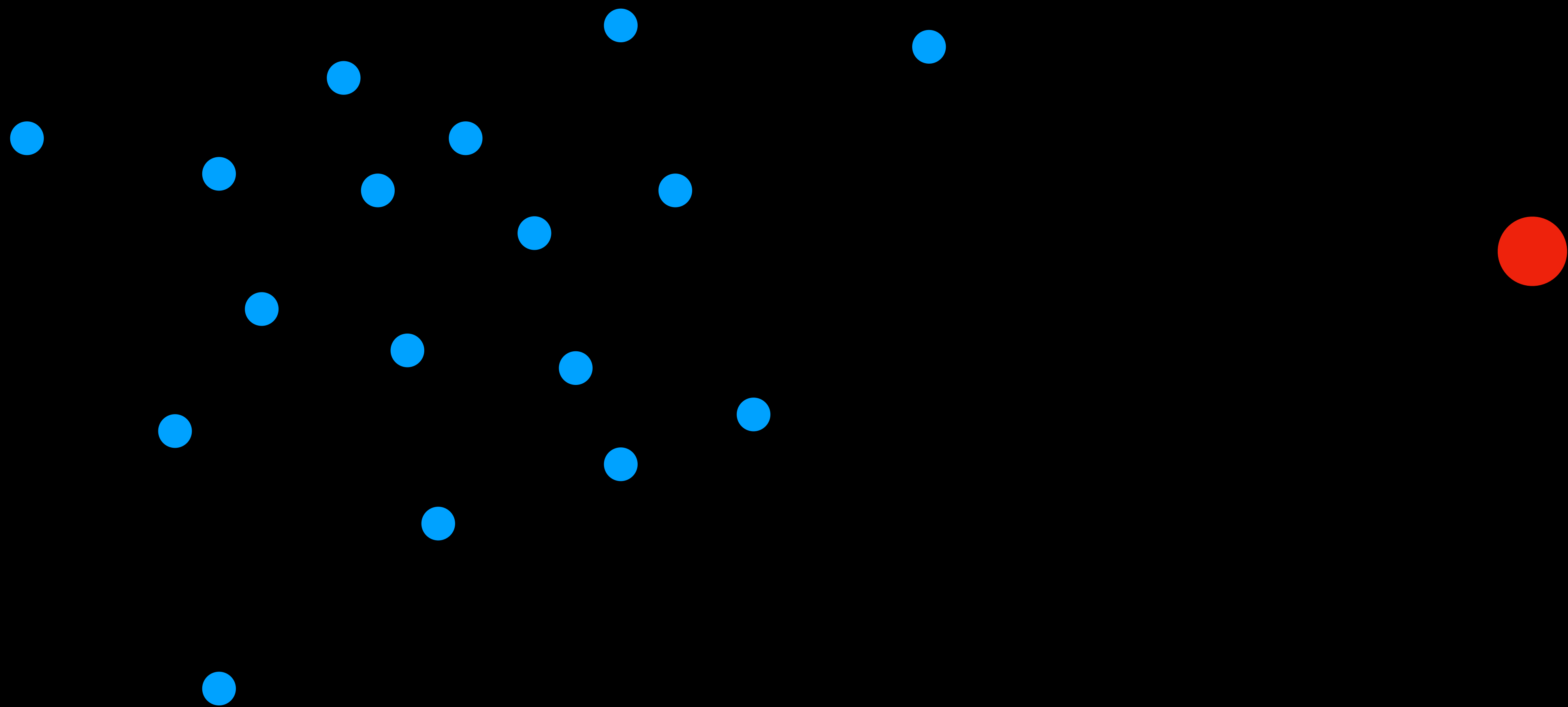
* Aging turns out to be not necessary.

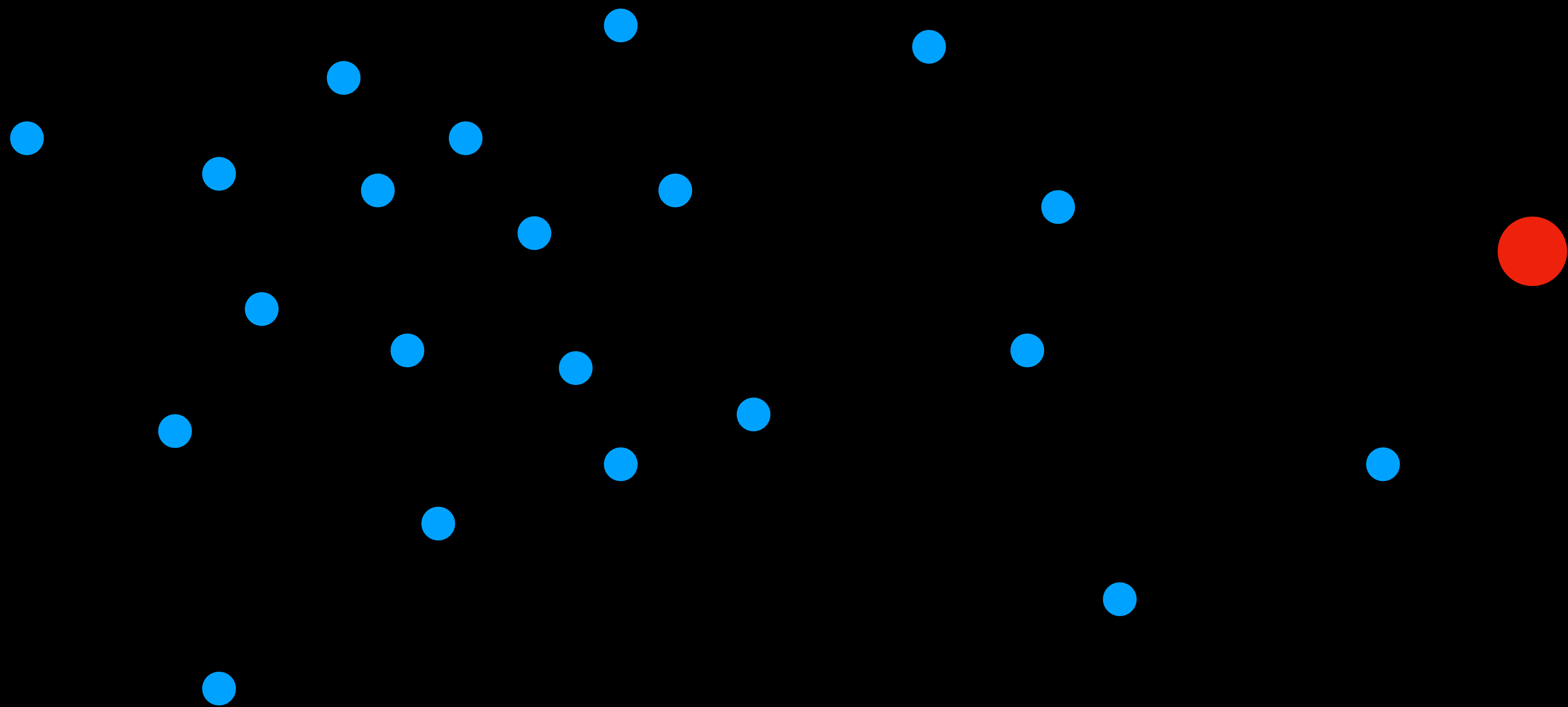


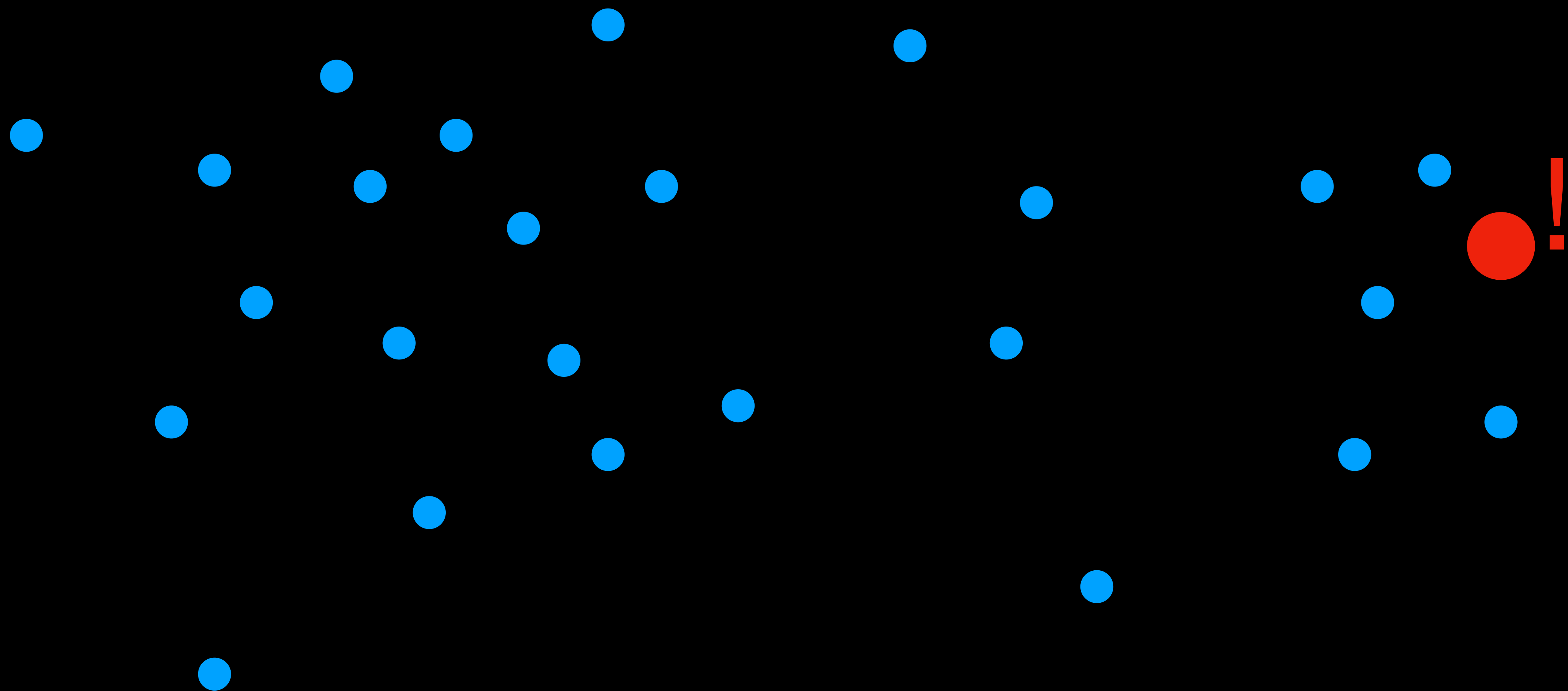






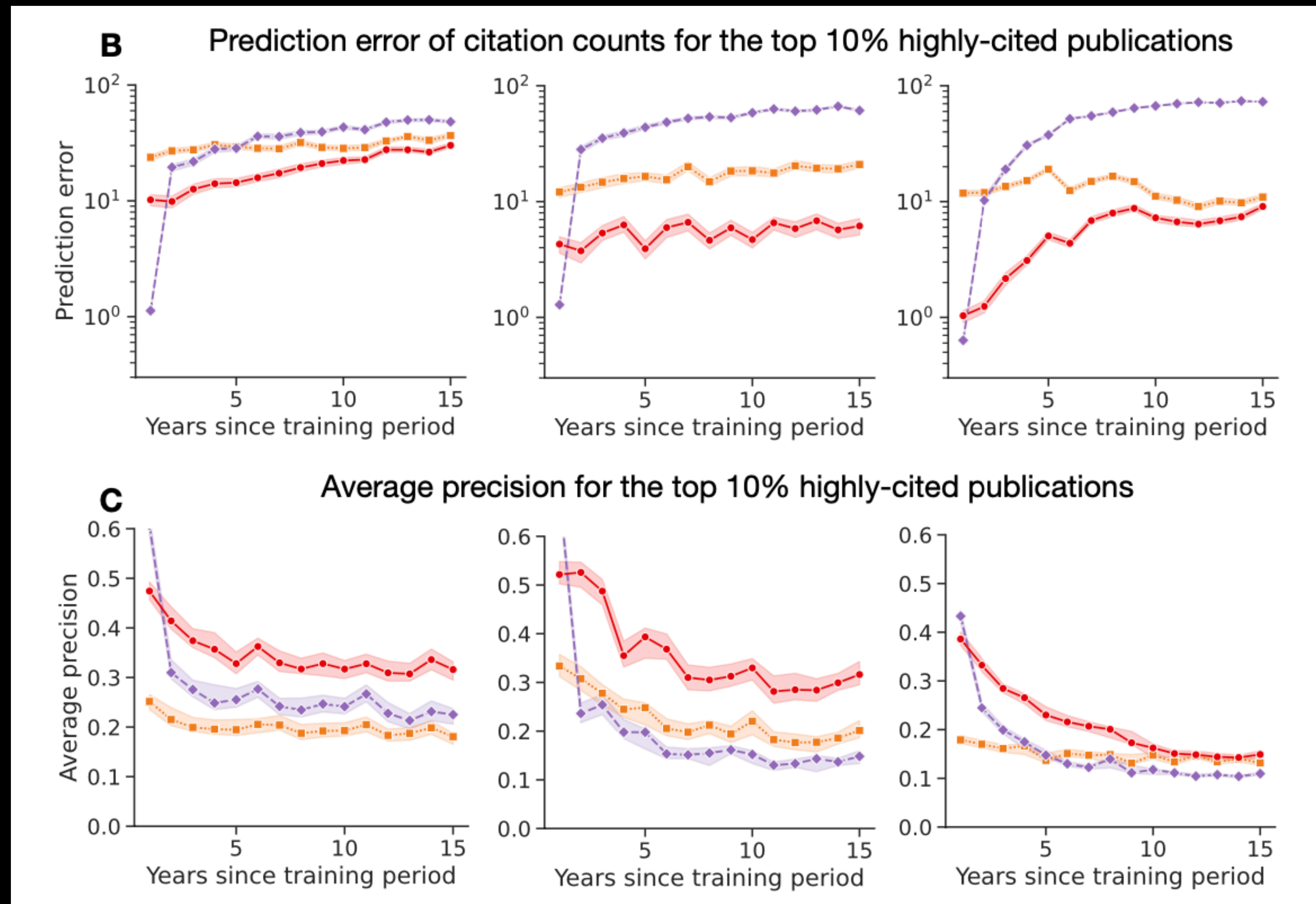






It allows us to “simulate”
the whole system.

It better predicts the future “hits”



Summary

- Citation-based knowledge systems exhibit many universal citation dynamics even with distinct modes of operations.
- Imagining the knowledge space allows us to shift the perspective from individual papers to the whole community.
- This shift lets us explain phenomena like sleeping beauties and better predict future citation patterns.

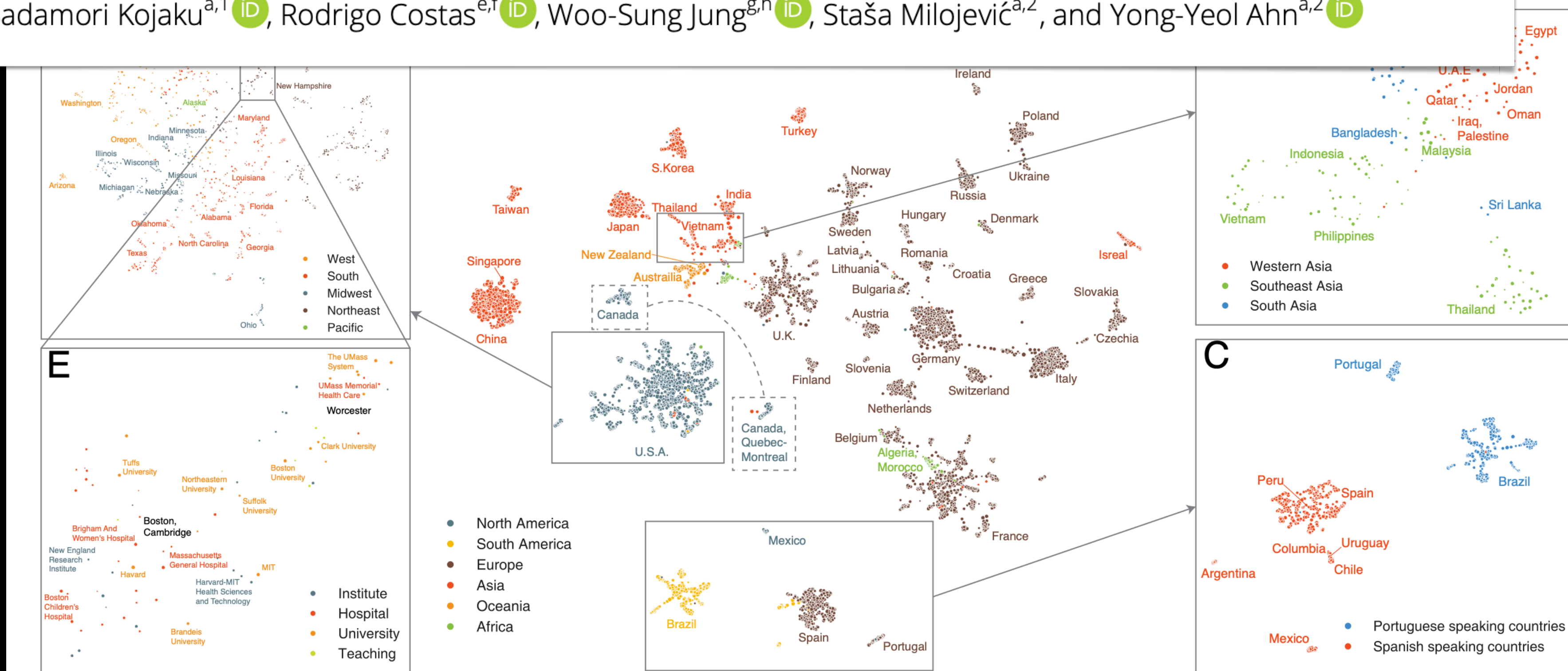


Can we concretely *interpret* the “distance”
in the knowledge embedding space?

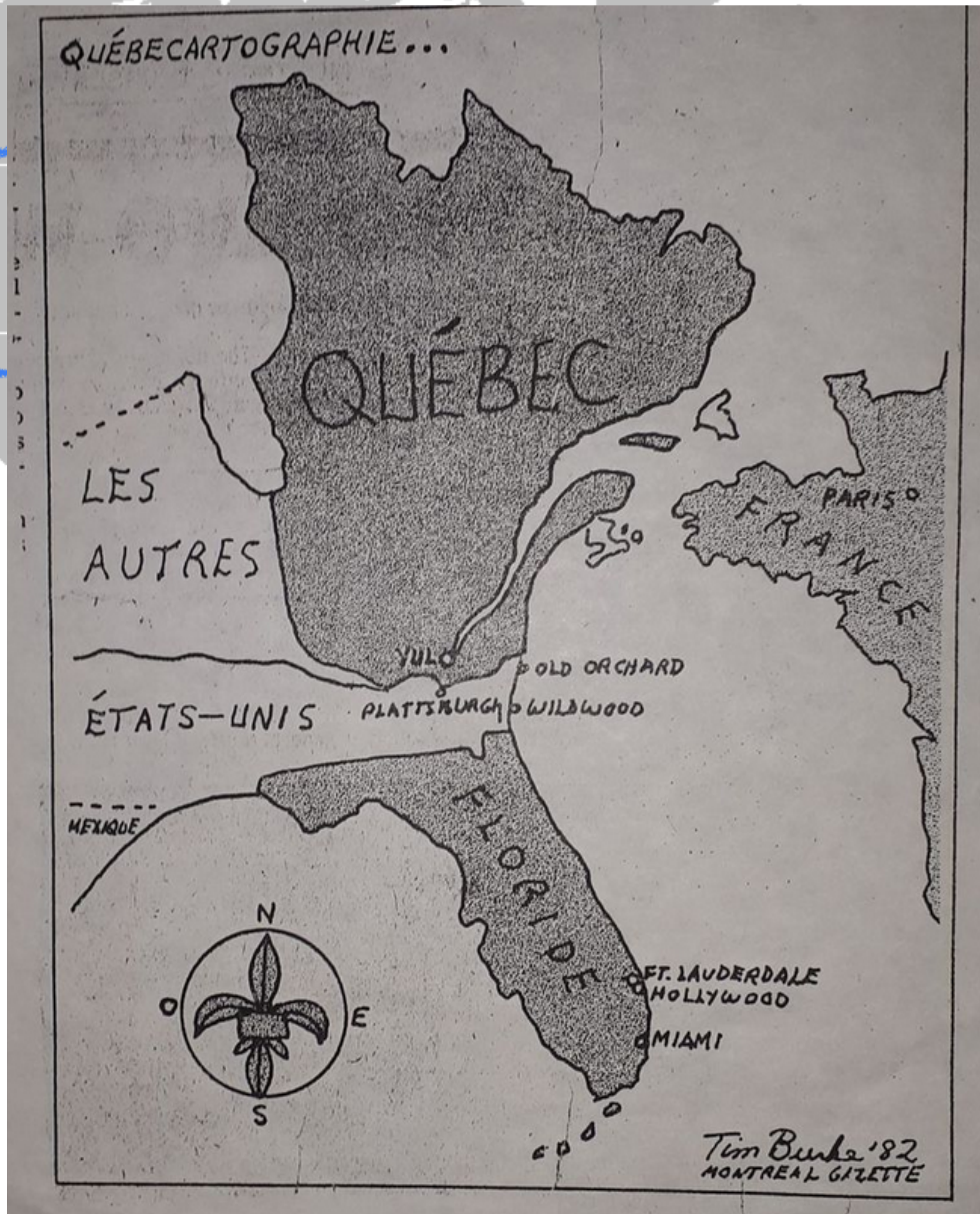


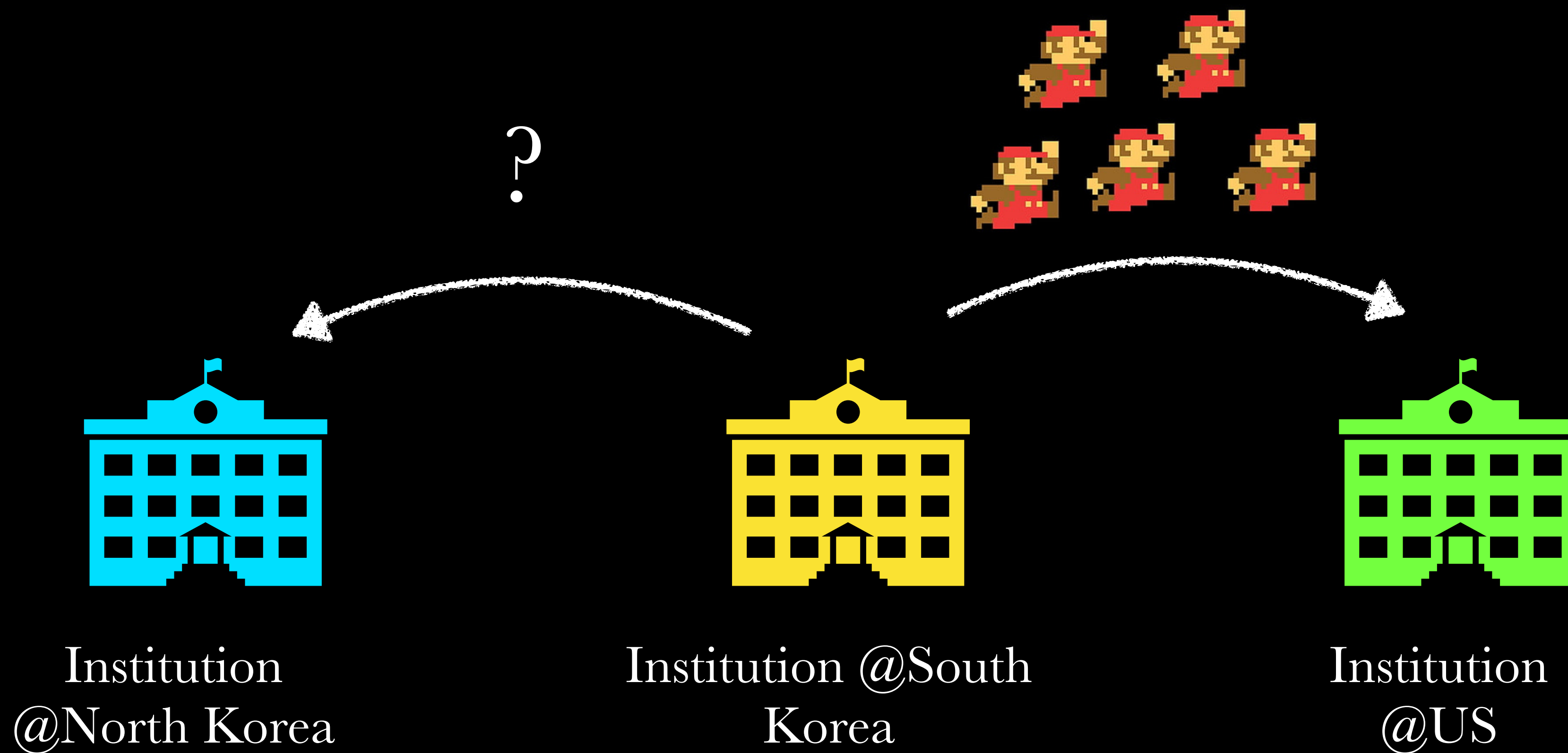
Unsupervised embedding of trajectories captures the latent structure of scientific migration

Dakota Murray^{a,b,1} , Jisung Yoon^{a,c,d,1}, Sadamori Kojaku^{a,1} , Rodrigo Costas^{e,f} , Woo-Sung Jung^{g,h} , Staša Milojević^{a,2}, and Yong-Yeol Ahn^{a,2}



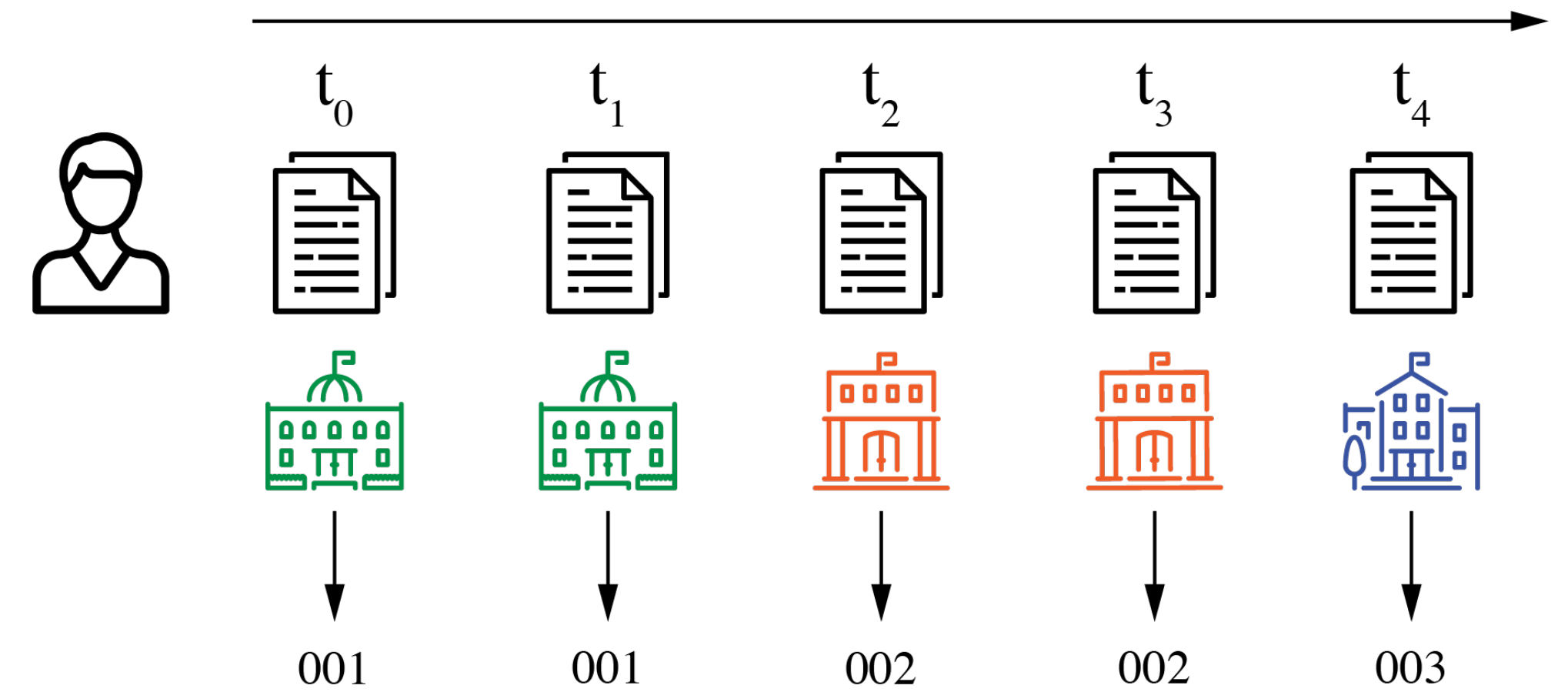




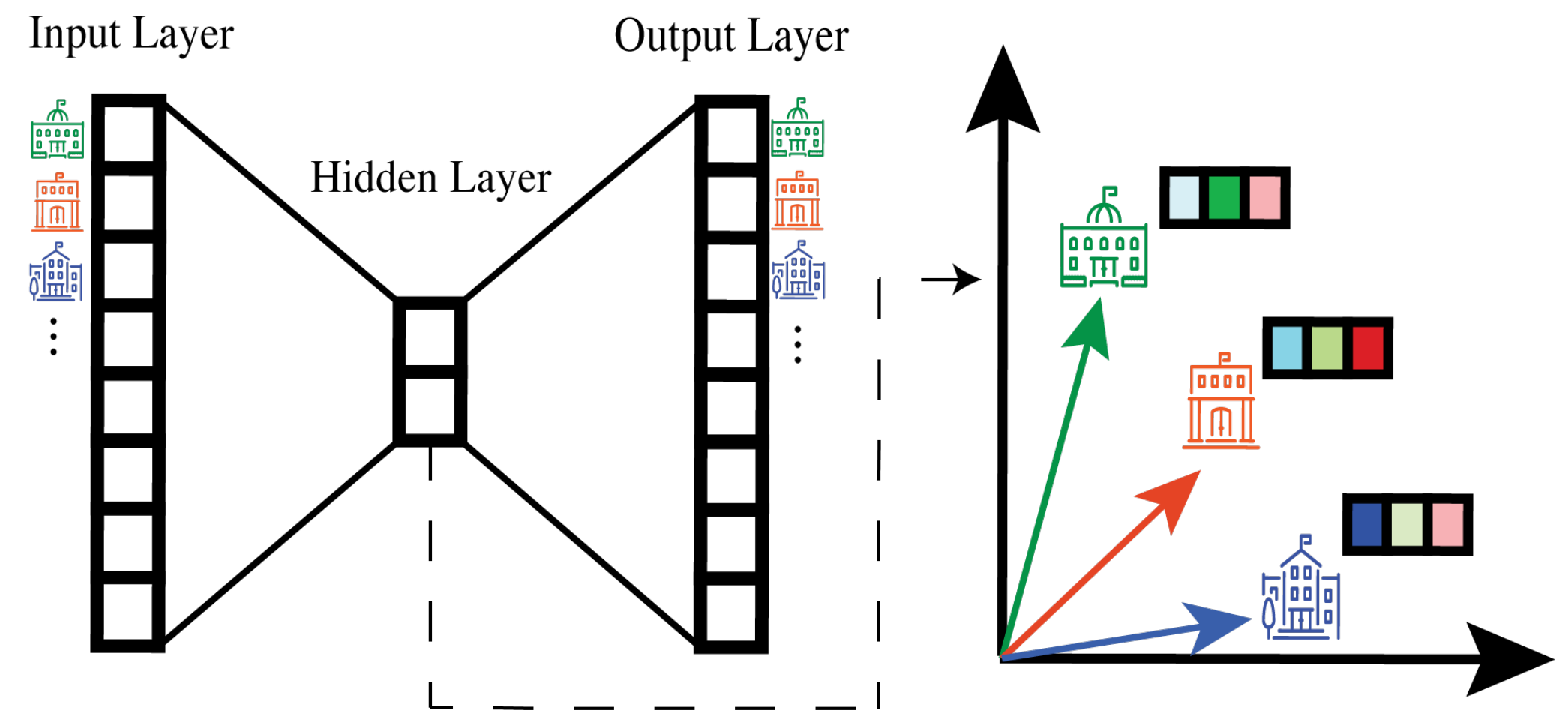


Social proximities do matter in human mobility

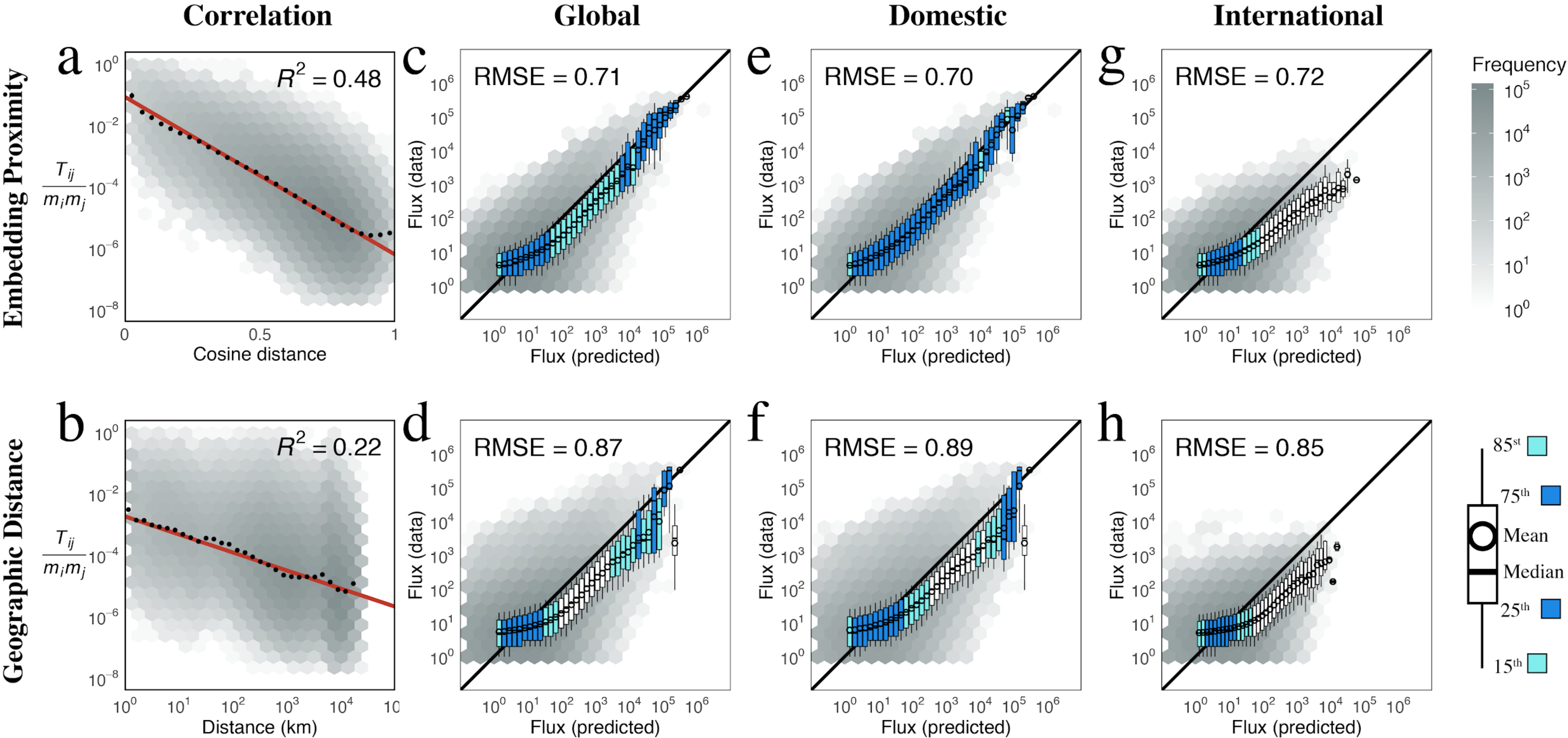
What happens if we use mobility of scientists to obtain the embedding?



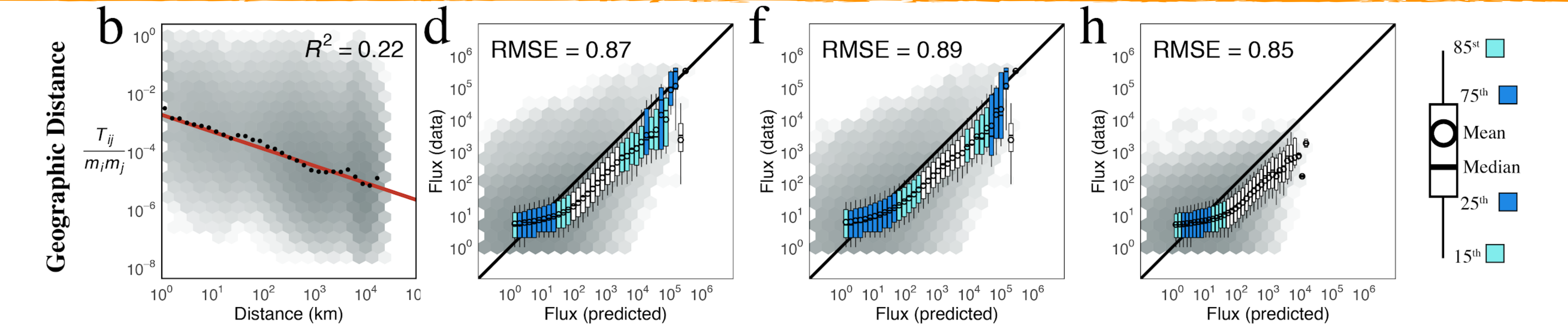
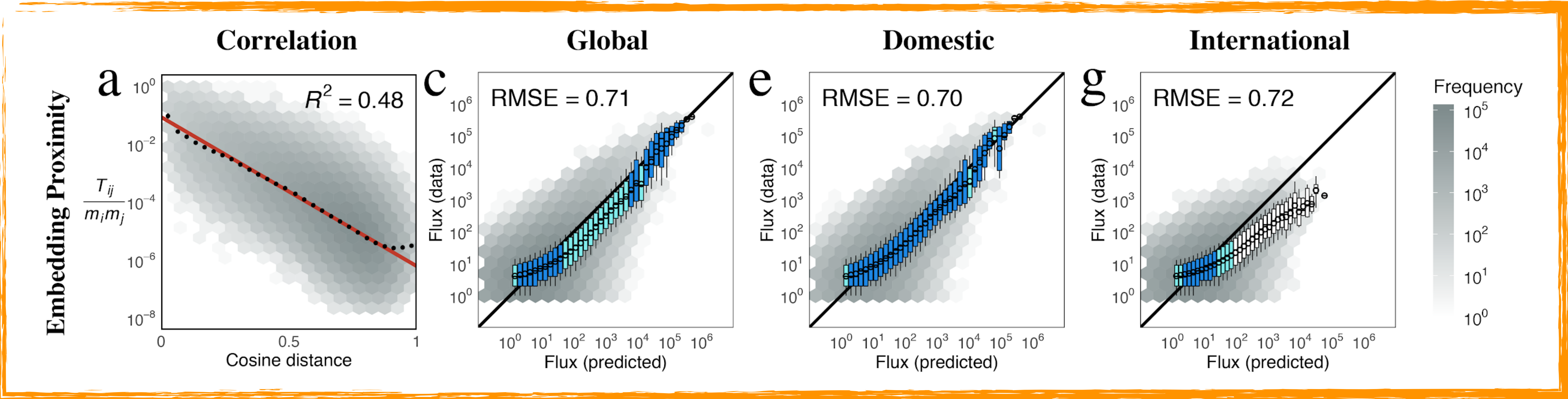
Trajectory: “001 — 001 — 002 — 002 — 003”



Embedding fits the gravity law much better!



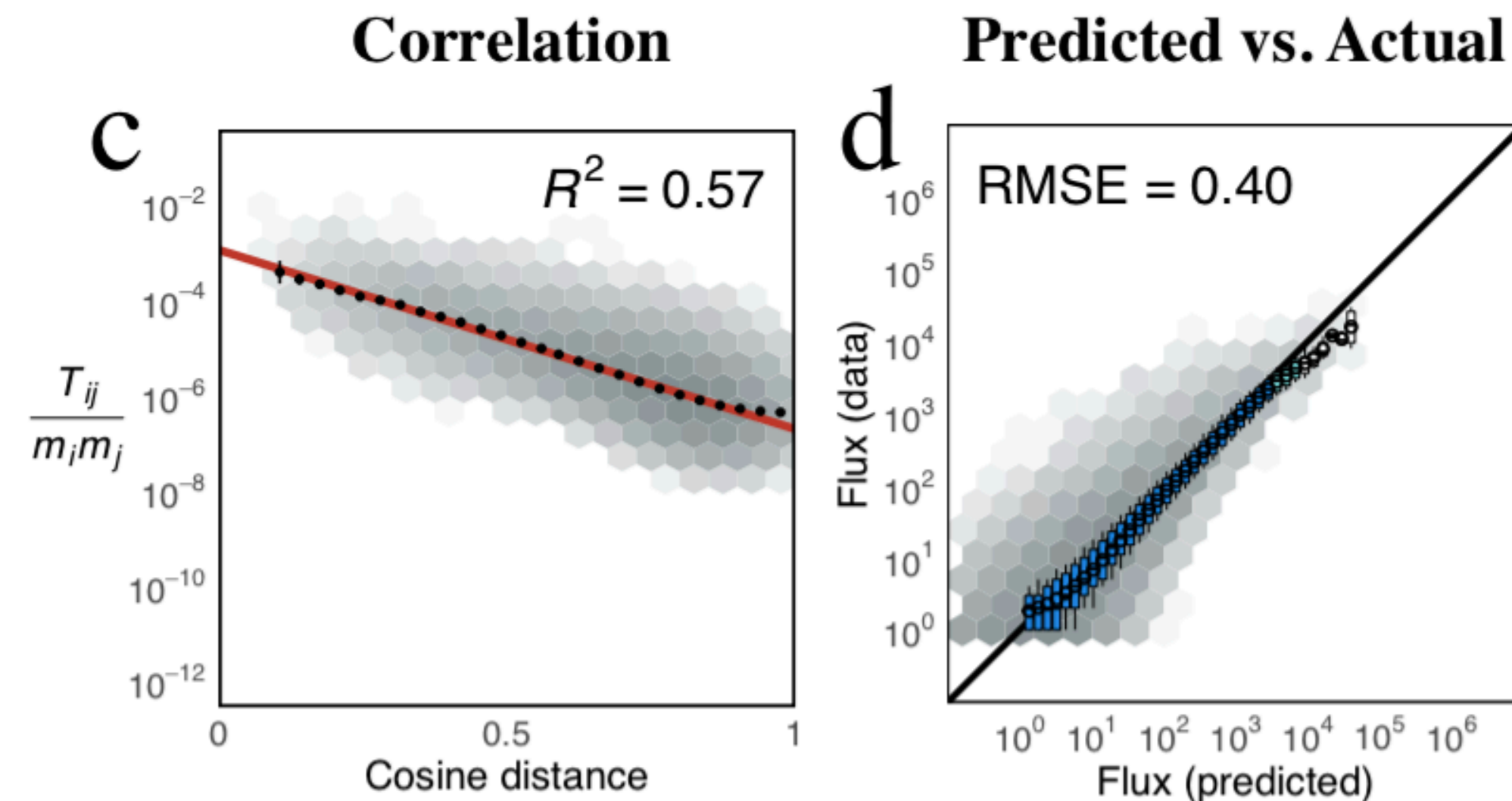
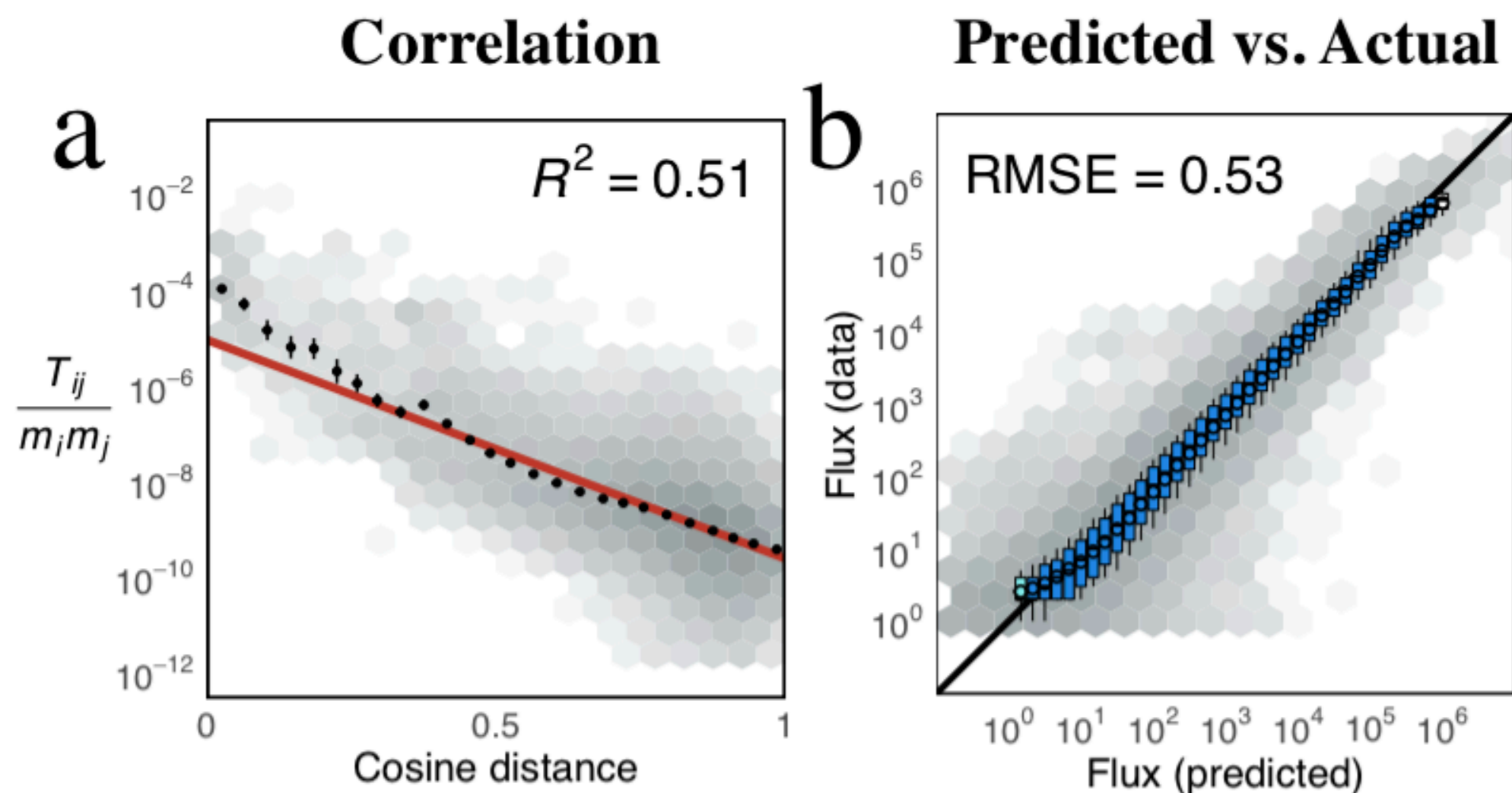
Embedding fits the gravity law much better!



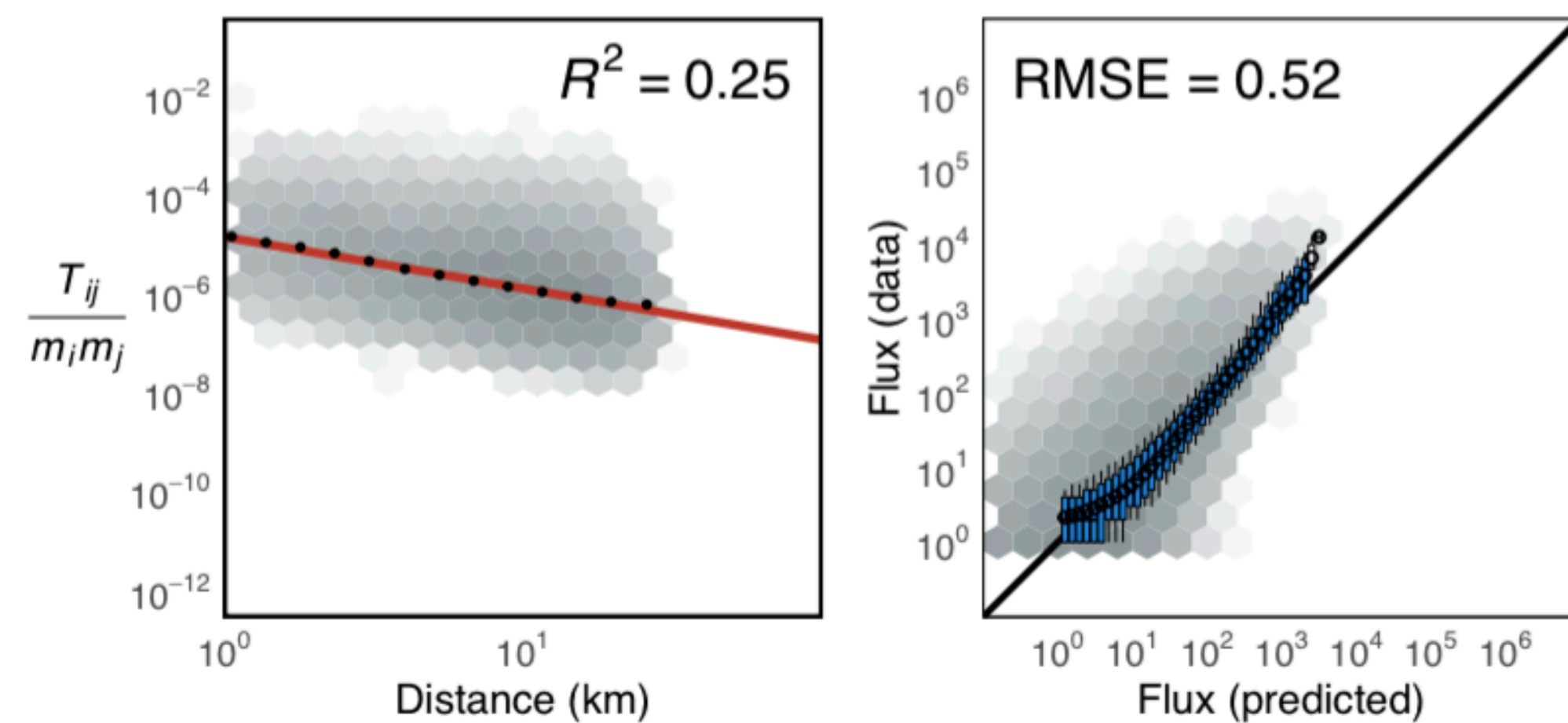
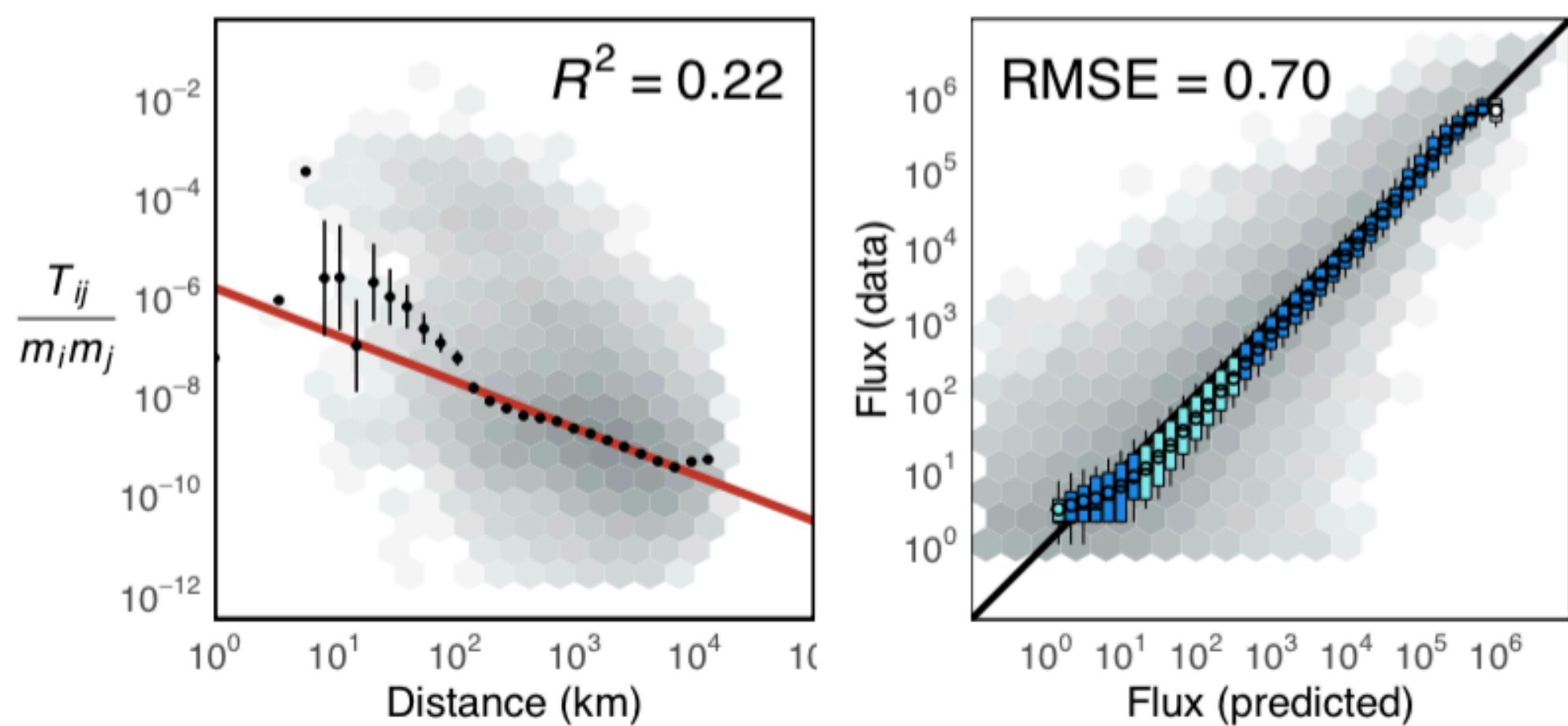
U.S. Flight Itineraries

Reservation

Embedding Proximity

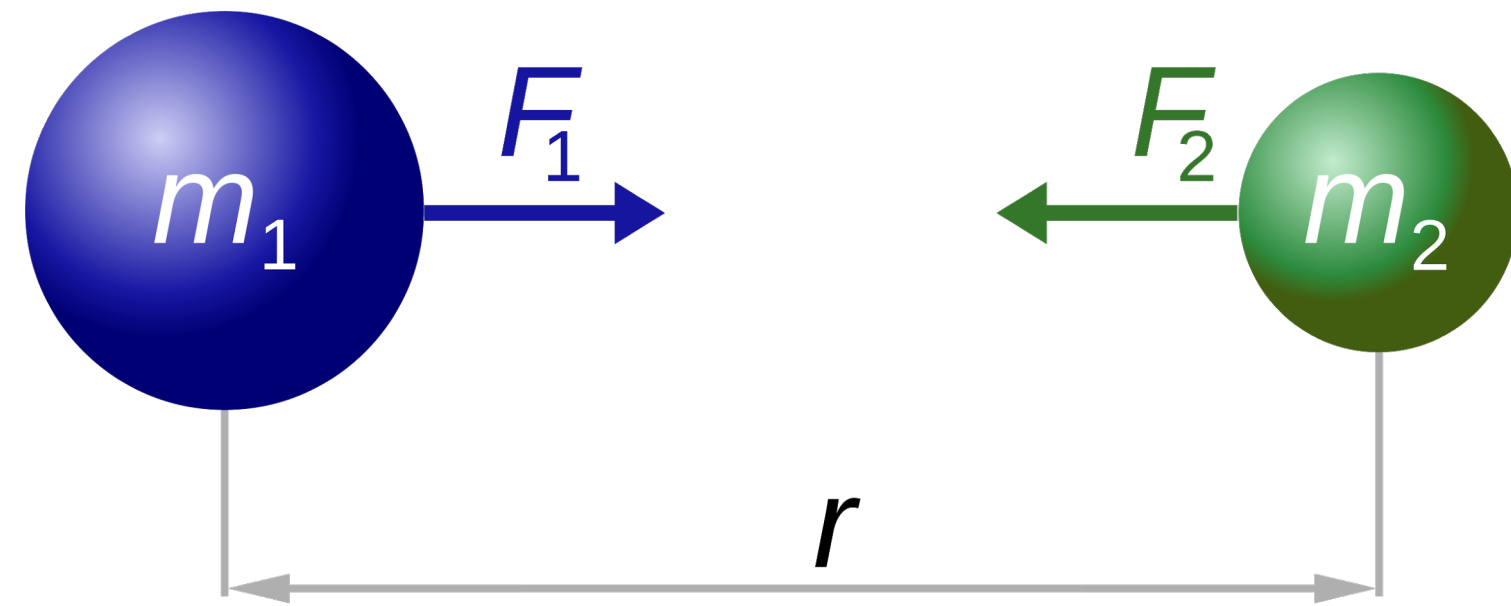


Geographic Distance



Gravity law of mobility

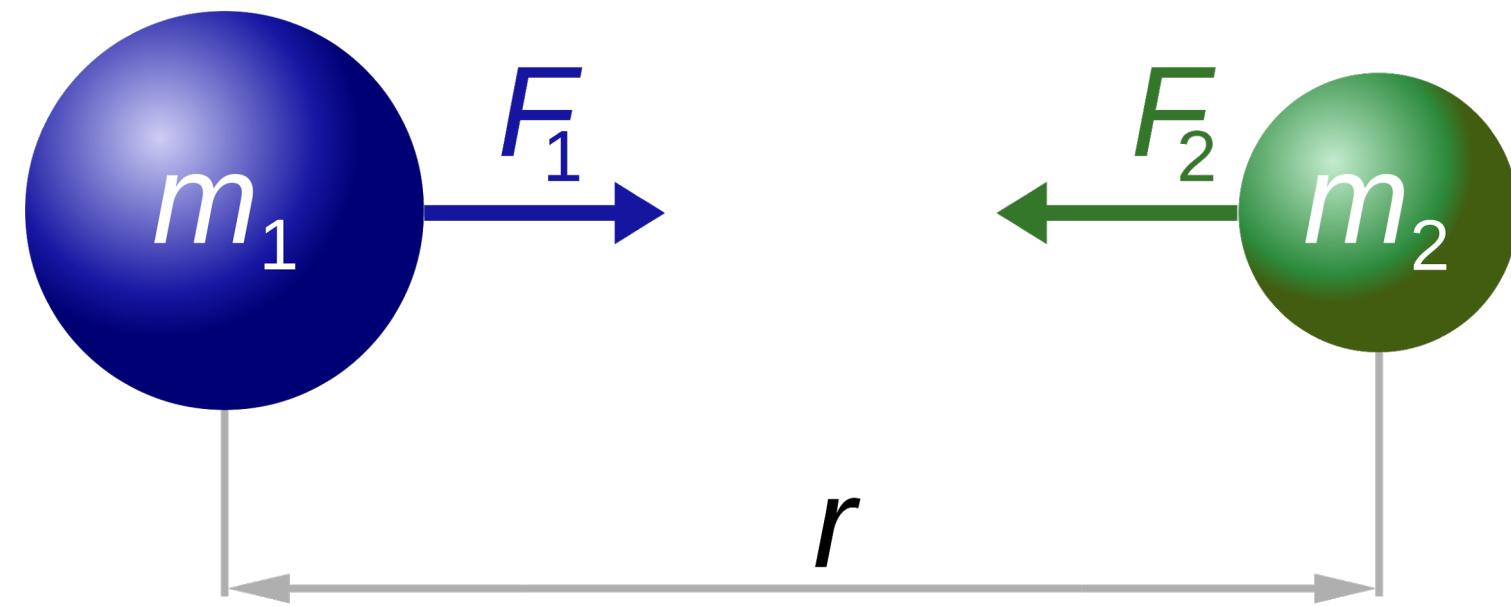
Gravity law of mobility



$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

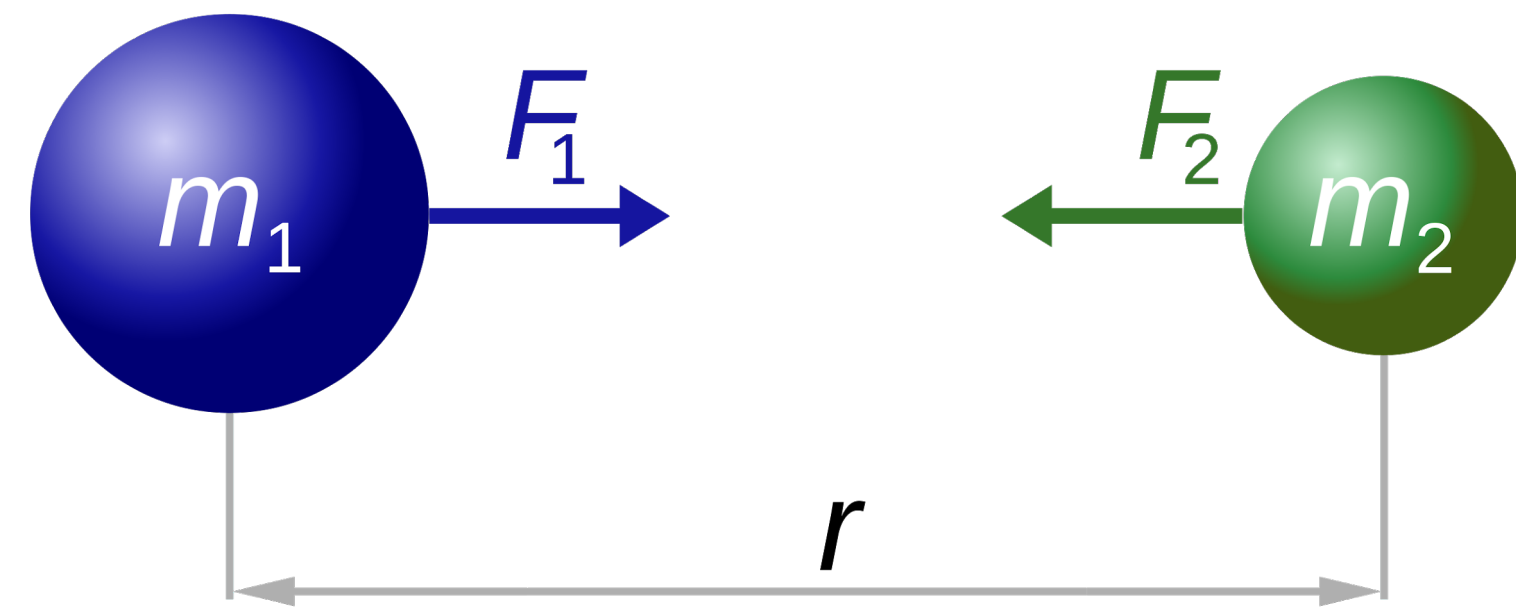
Gravity law of mobility

“You are less likely to go somewhere farther away than somewhere close.”



$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

Gravity law of mobility

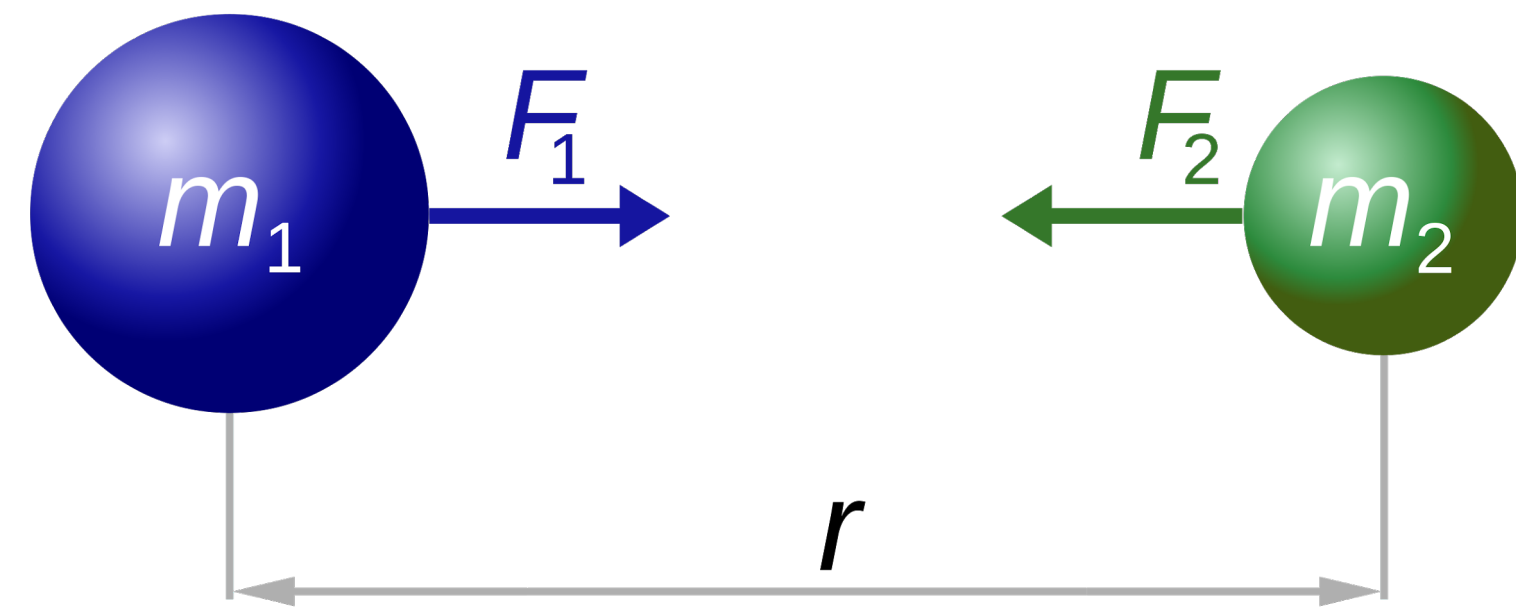


$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

“You are less likely to go somewhere farther away than somewhere close.”

$$\hat{T}_{ij} = C m_i m_j f(r_{ij})$$

Gravity law of mobility

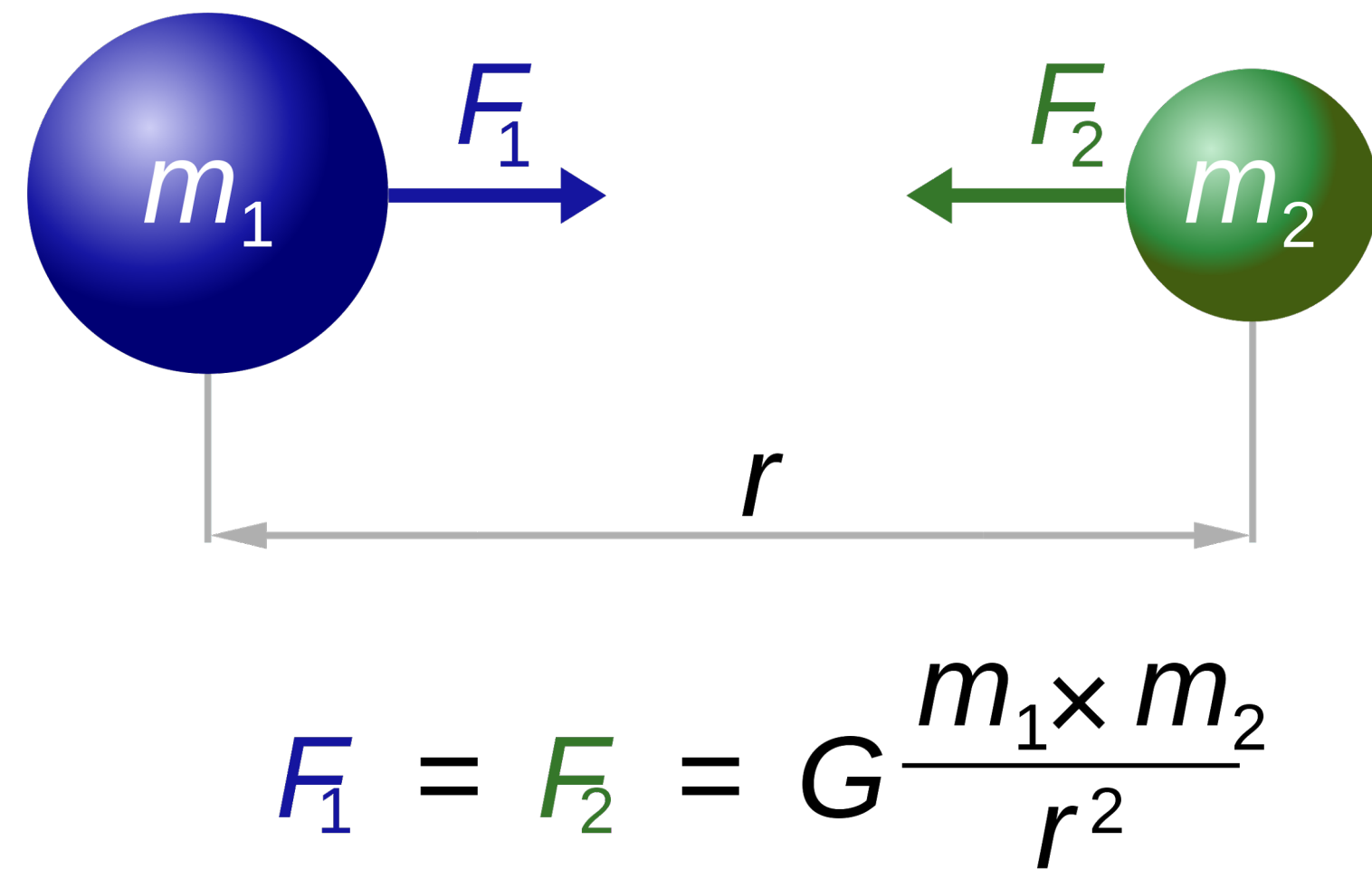


$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

“You are less likely to go somewhere farther away than somewhere close.”

Flux $\nearrow \hat{T}_{ij} = C m_i m_j f(r_{ij})$

Gravity law of mobility



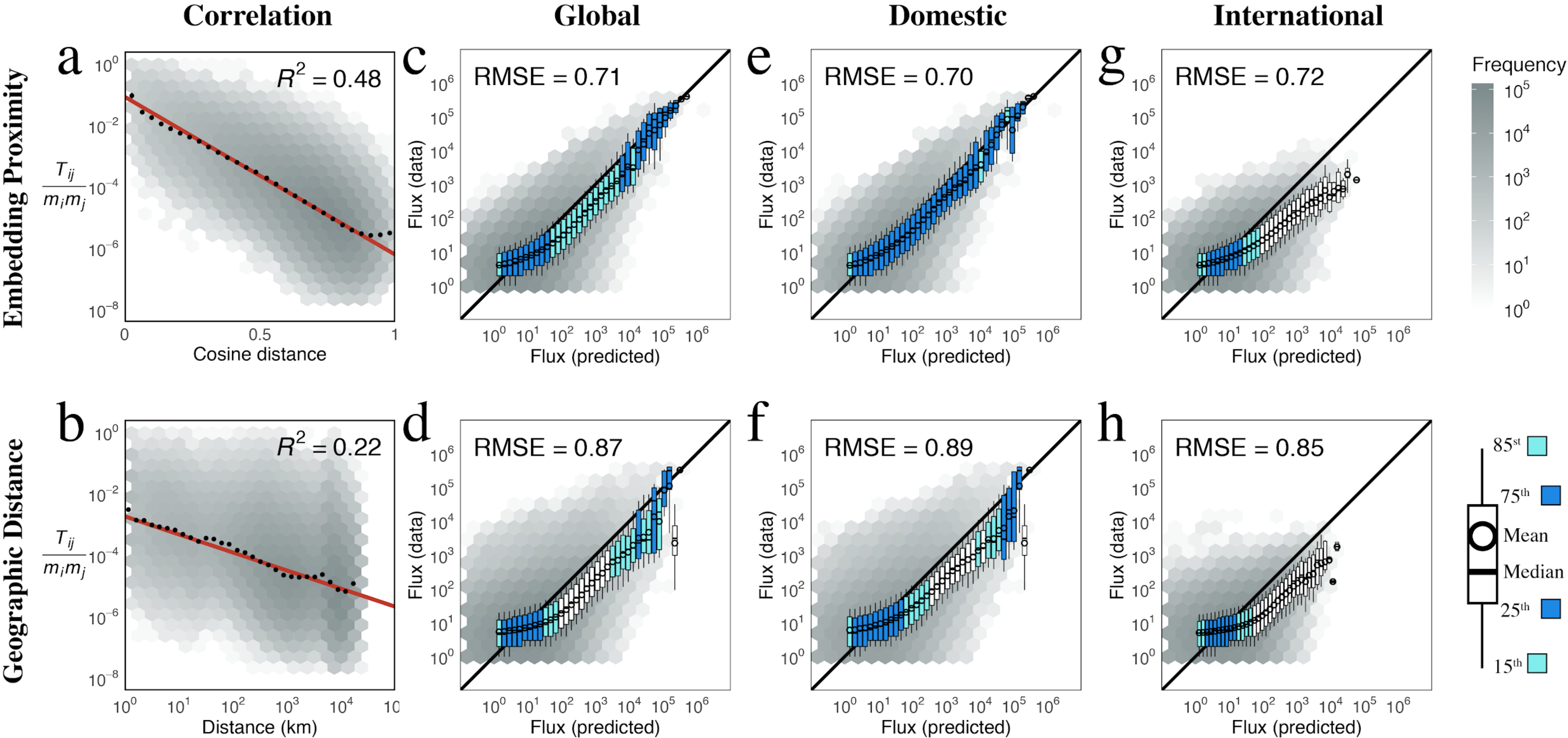
“You are less likely to go somewhere farther away than somewhere close.”

$$\hat{T}_{ij} = C m_i m_j \underbrace{f(r_{ij})}_{\text{a decay function}}$$

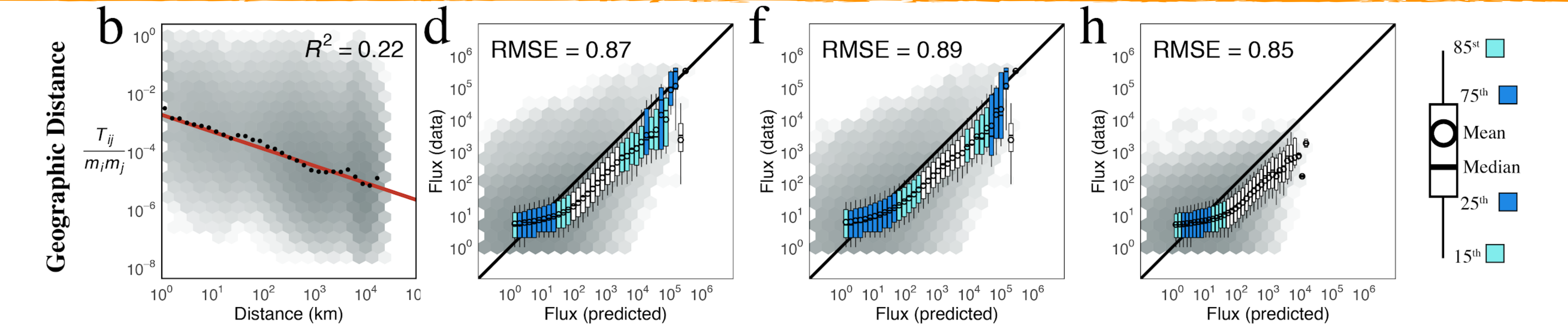
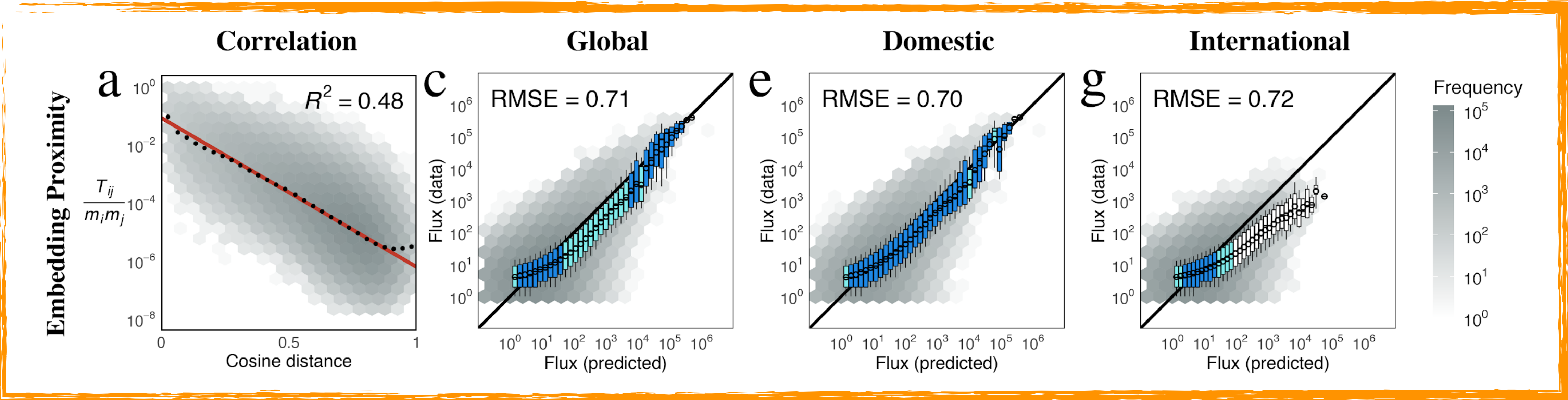
Flux

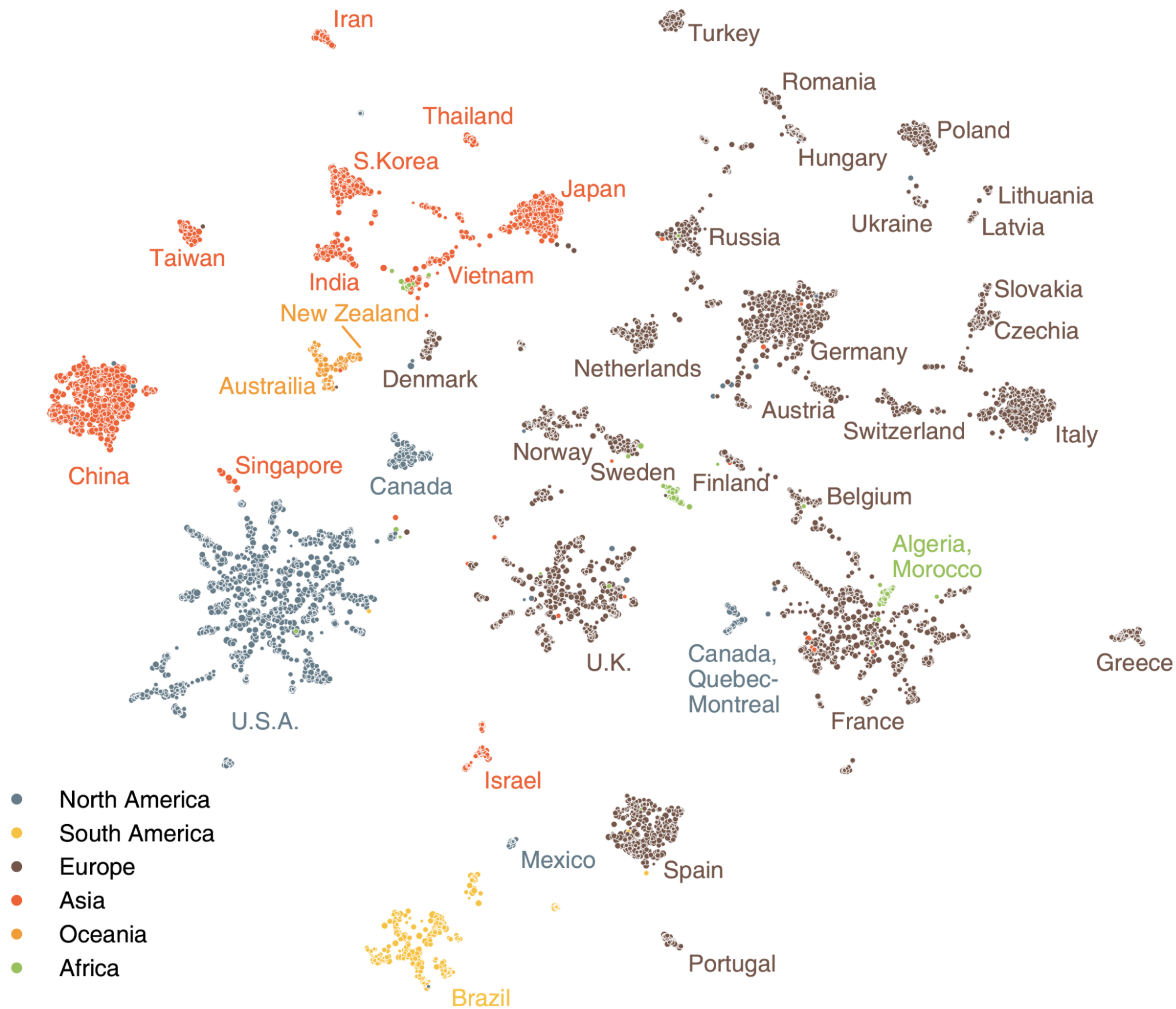
Population

Embedding fits the gravity law much better!

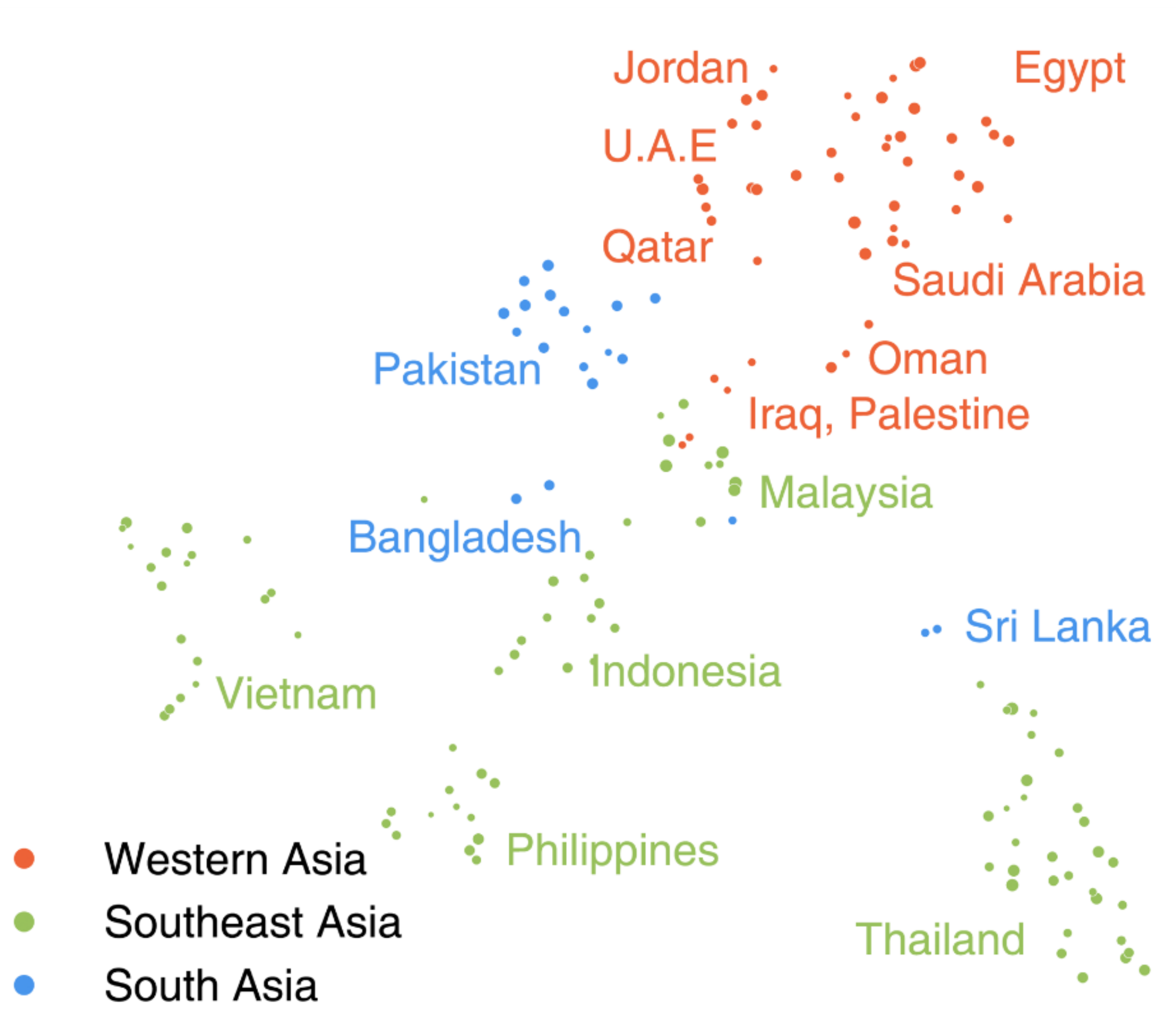


Embedding fits the gravity law much better!

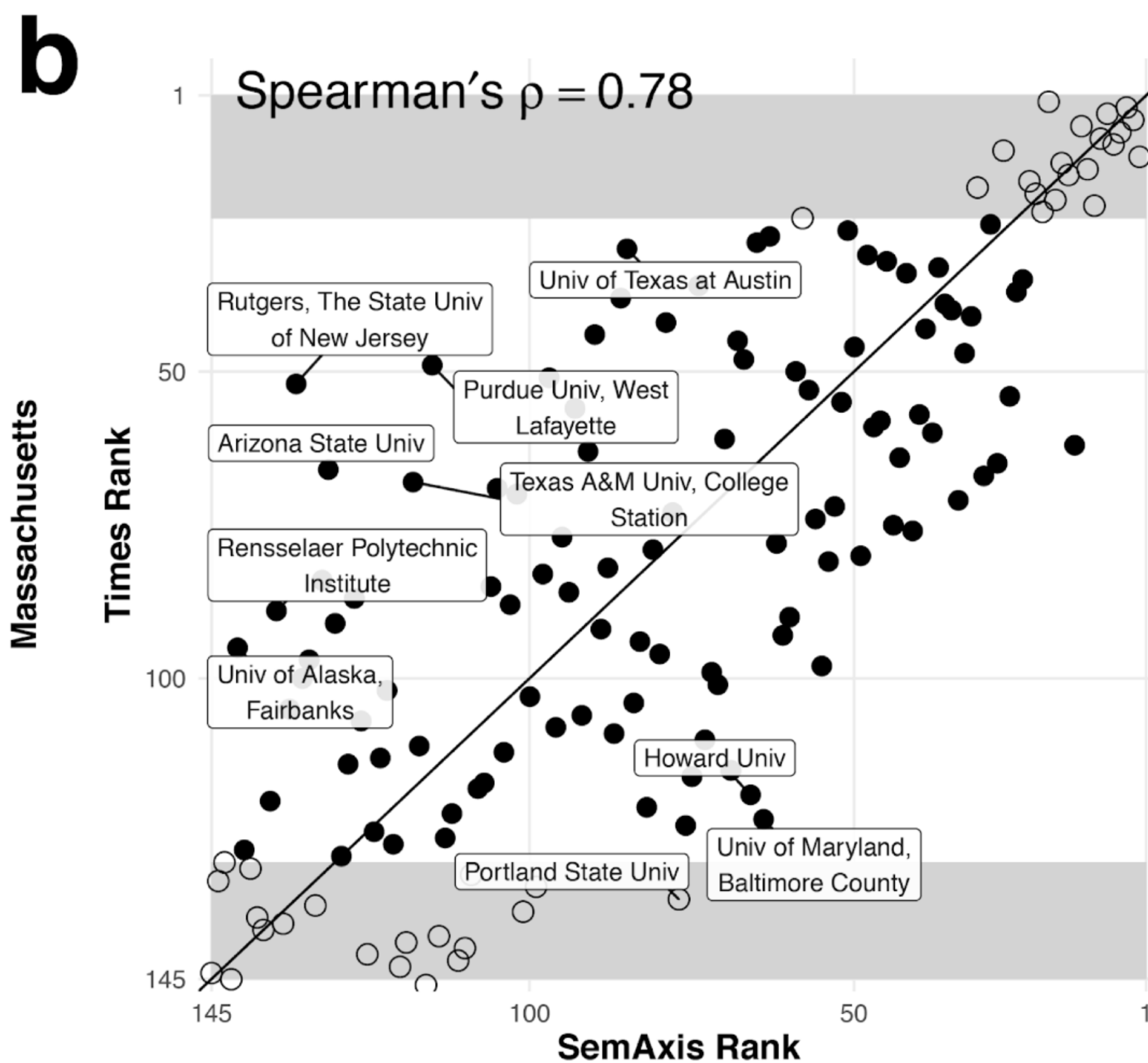




It encodes regional & cultural structures



It encodes prestige

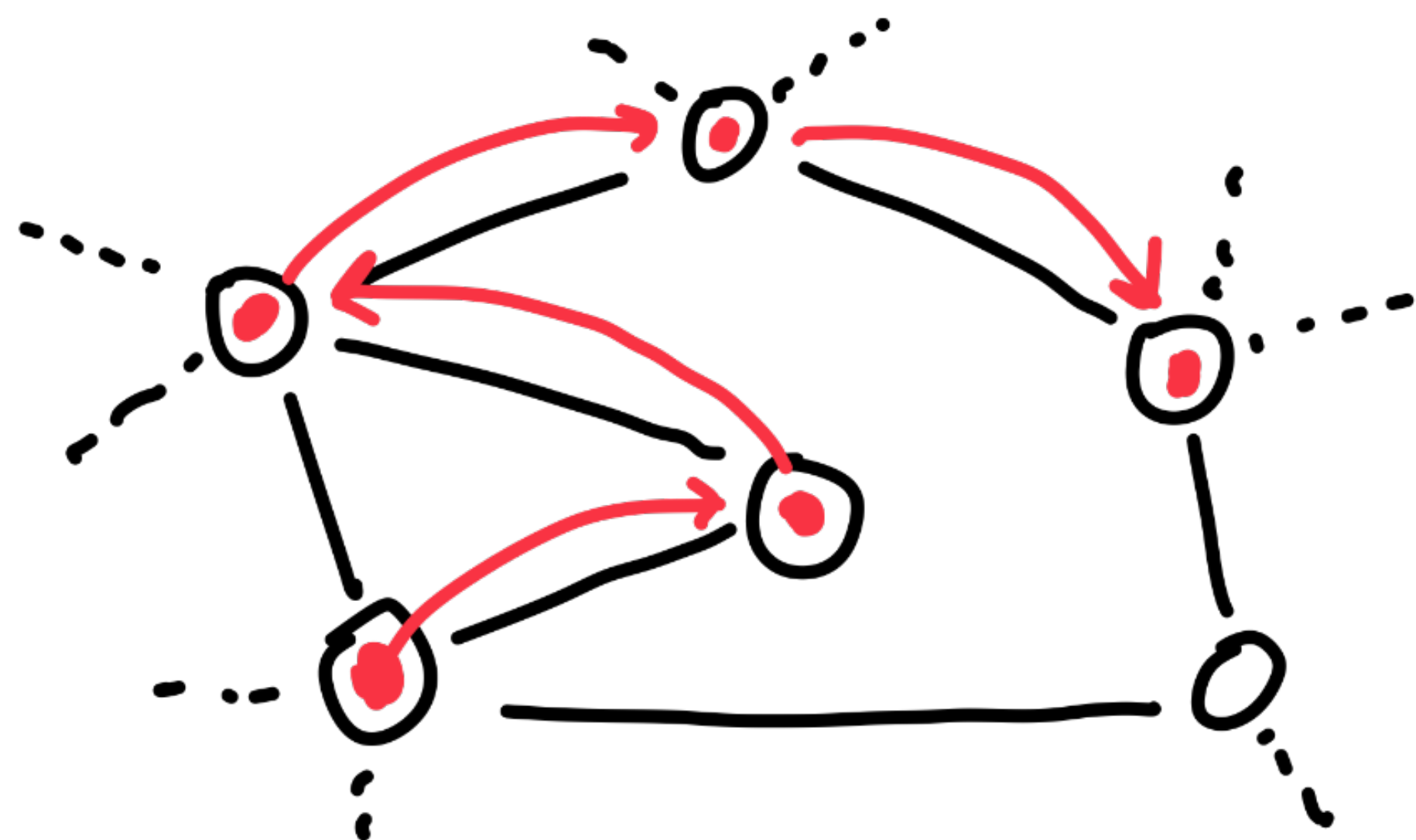


Why?



But most surprisingly, we found that,
with some maths...

$$P(w_t | w_c) \approx \frac{p_n(t) \exp(\mathbf{k}_t \cdot \mathbf{q}_c)}{\sum_i p_n(i) \exp(\mathbf{k}_i \cdot \mathbf{q}_c)}$$



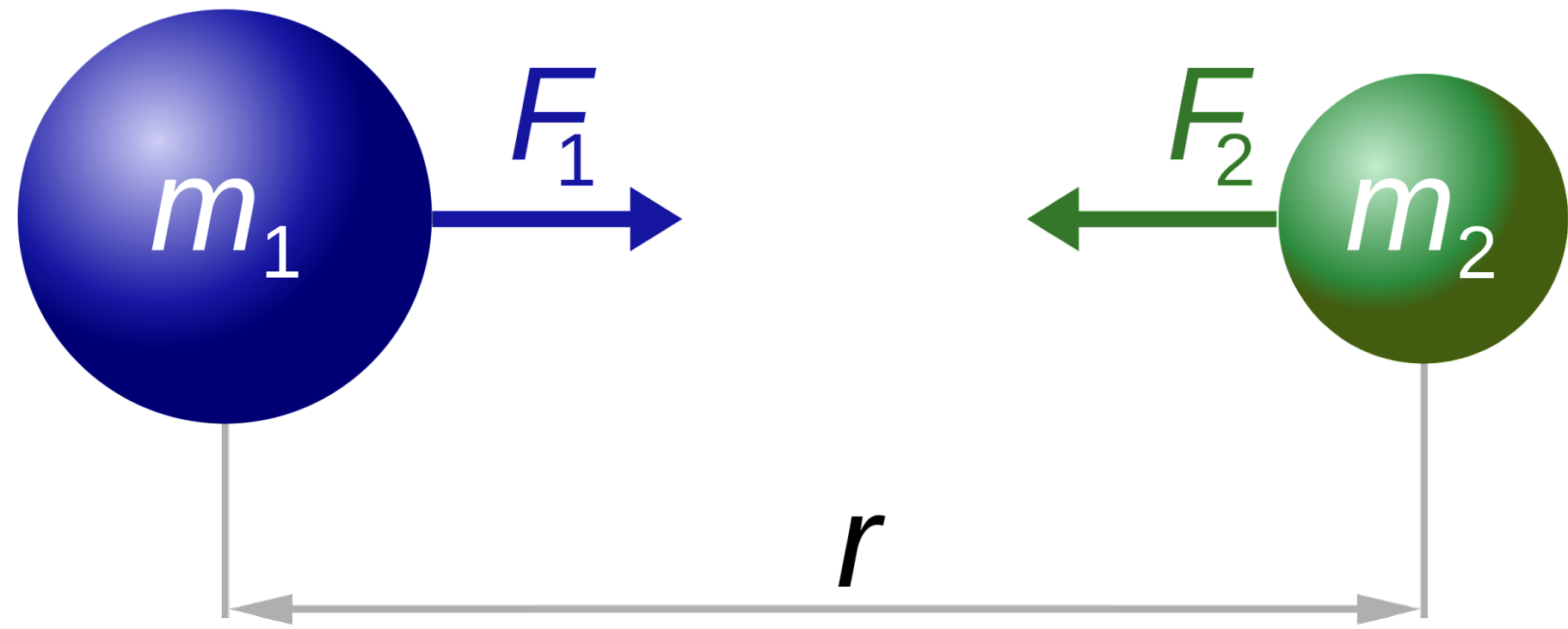
flux

$$\hat{T}_{ij} \propto P(j | i) P(i) \propto \frac{P(i) P(j) \exp(\mathbf{k}_j \cdot \mathbf{q}_i)}{\sum_{j'} P(j') \exp(\mathbf{k}_{j'} \cdot \mathbf{q}_i)}$$

when embedding dimension is sufficiently large

$$\hat{T}_{ij} = \hat{T}_{ji} \propto P(i) P(j) \exp(\mathbf{k}_i \cdot \mathbf{k}_j) \rightarrow \text{Gravity law!}$$

Word2vec model is equivalent to the **Gravity law of mobility!**



$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

Wikipedia user [Dennis Nilsson](#)

$$\hat{T}_{ij} = C m_i m_j f(r_{ij})$$

Flux

Population

A decay function

The diagram shows the equation $\hat{T}_{ij} = C m_i m_j f(r_{ij})$. Arrows point from the labels below to the terms in the equation: 'Flux' points to $\hat{T}_{ij}$, 'Population' points to $m_i m_j$, and 'A decay function' points to $f(r_{ij})$.

word2vec model ~ gravity law



The embedding distance has a concrete, interpretable meaning.

The space where the institutions are arranged so that the flux and distance between them satisfies the gravity law of mobility!

Interestingly...



Gravity law of embedding

$$F = G \frac{m_1 m_2}{r^2}$$

Mass

$$m_i \sim P(i)$$

Closeness

$$\frac{1}{r^2} \sim \exp(u_i^\top v_j)$$

This law is model agnostic!

Gravity law of embedding

$$F = G \frac{m_1 m_2}{r^2}$$

Mass

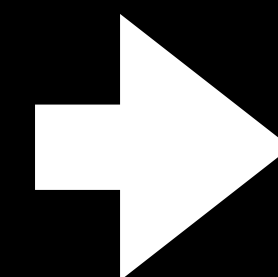
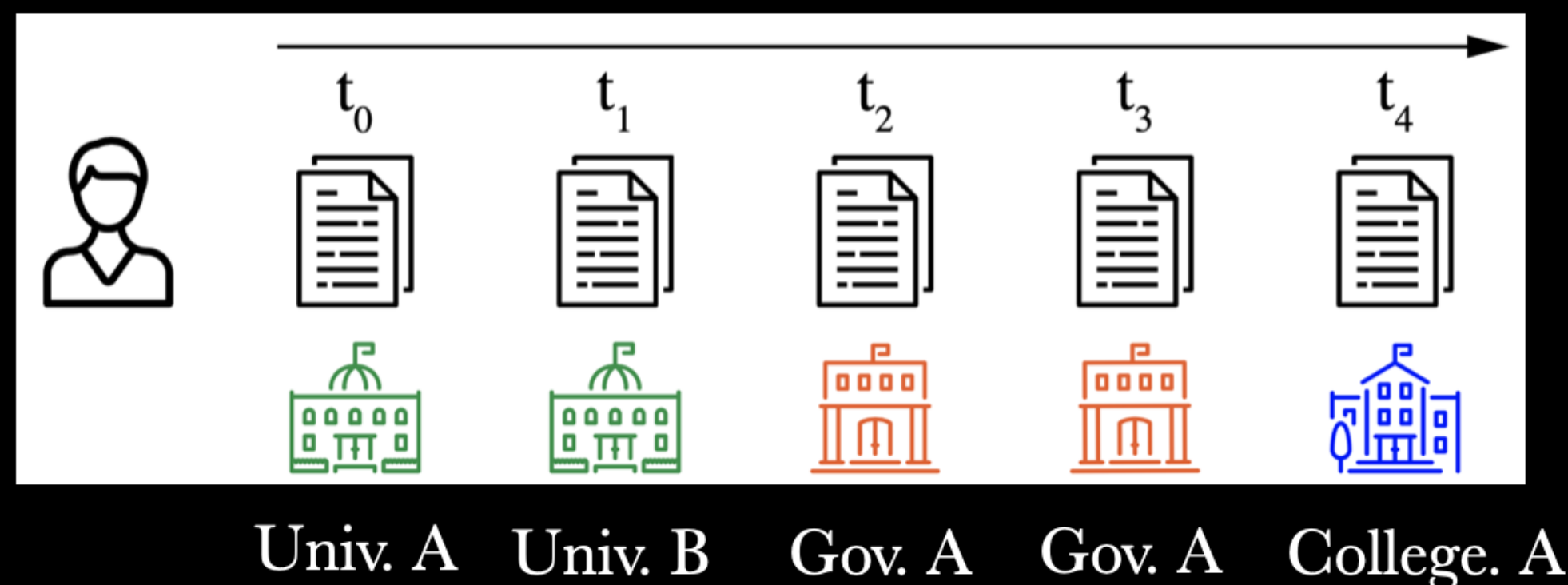
$$m_i \sim P(i)$$

Closeness

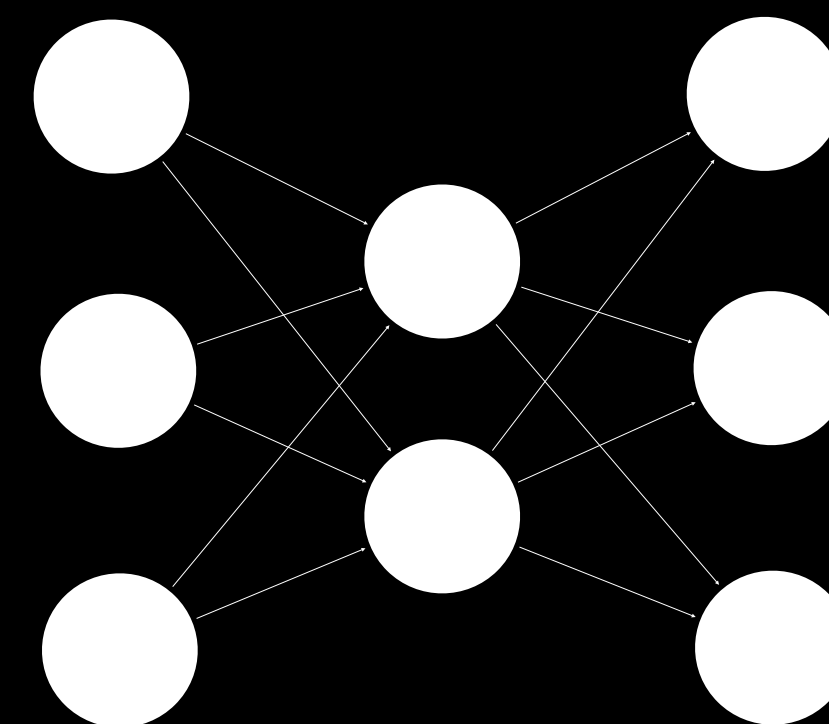
$$\frac{1}{r^2} \sim \exp(u_i^\top v_j)$$

This law is model agnostic!

Affiliation trajectories



word2vec



Gravity law of embedding

$$F = G \frac{m_1 m_2}{r^2}$$

Mass

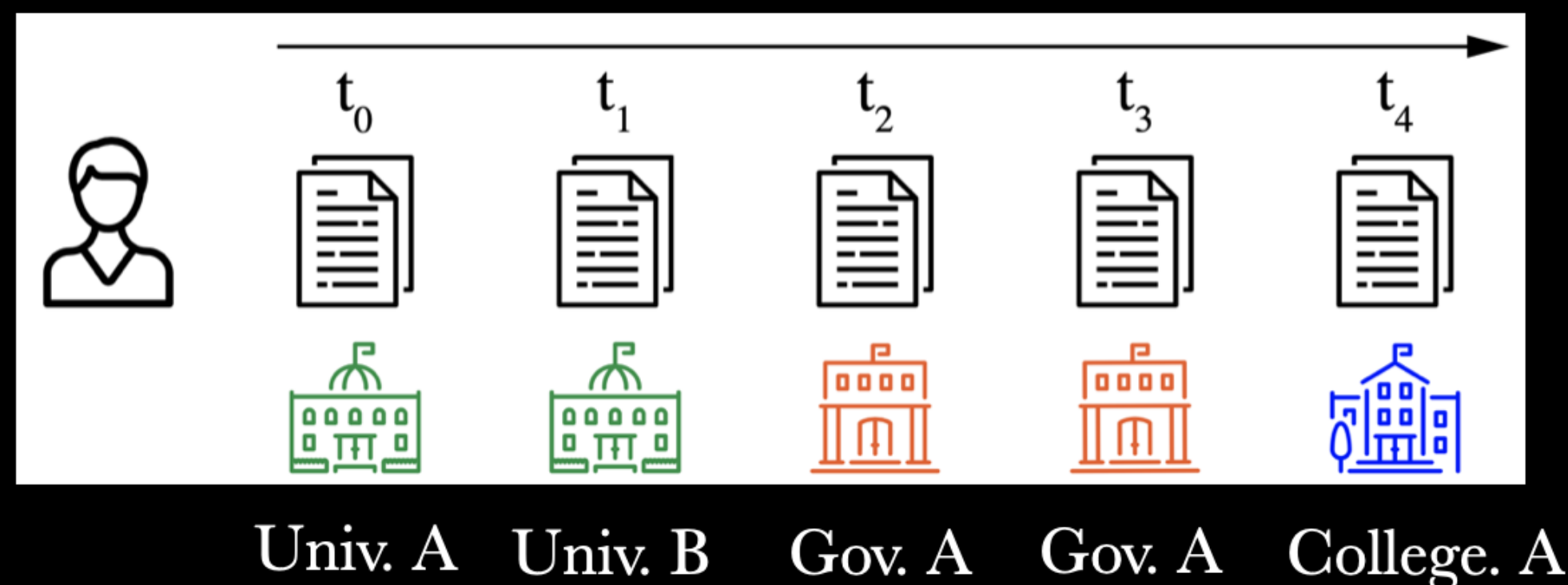
$$m_i \sim P(i)$$

Closeness

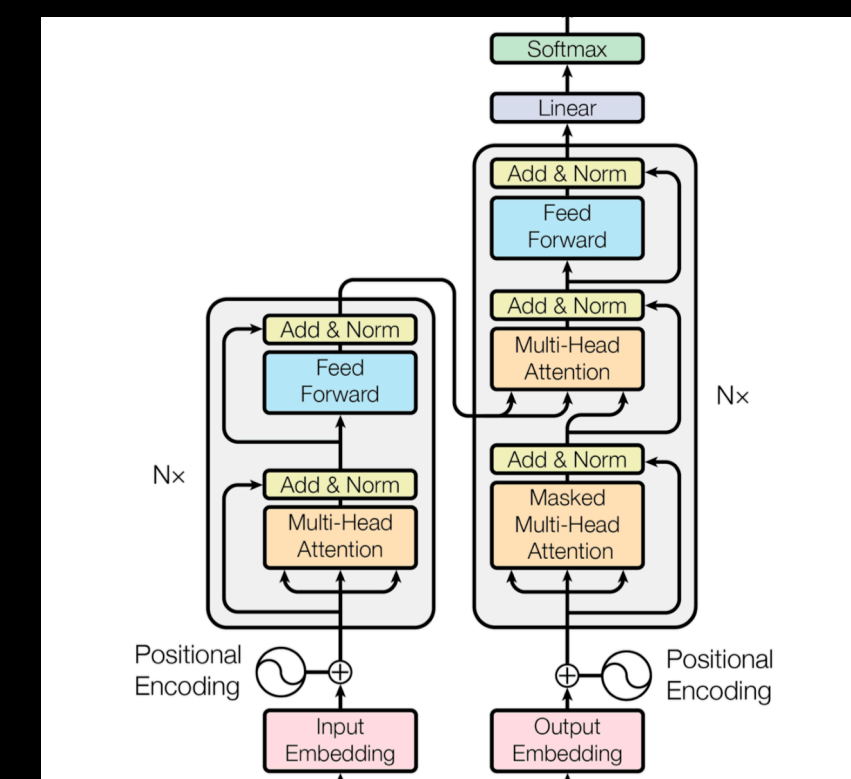
$$\frac{1}{r^2} \sim \exp(u_i^\top v_j)$$

This law is model agnostic.

Affiliation trajectories



Transformers



Gravity law of embedding

$$F = G \frac{m_1 m_2}{r^2}$$

Mass

$$m_i \sim P(i)$$

Closeness

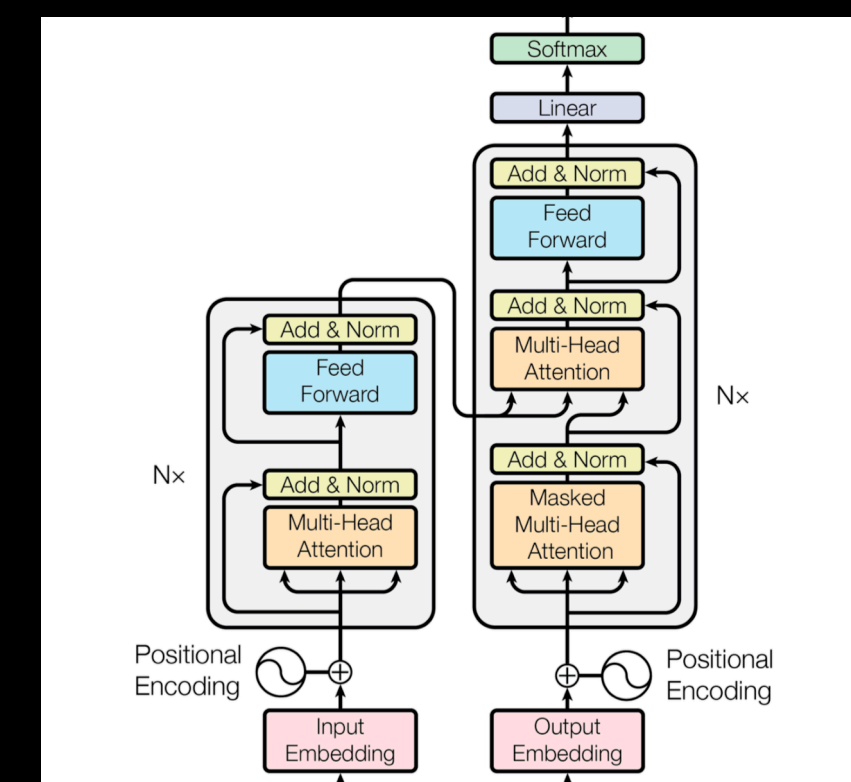
$$\frac{1}{r^2} \sim \exp(u_i^\top v_j)$$

This law is model agnostic.

Paper text trajectories

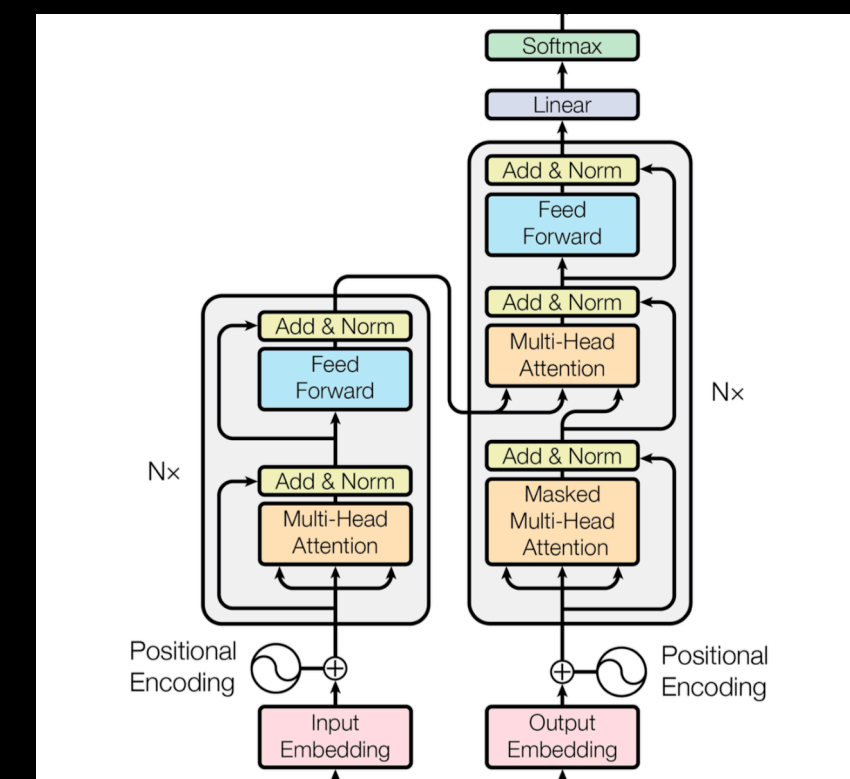
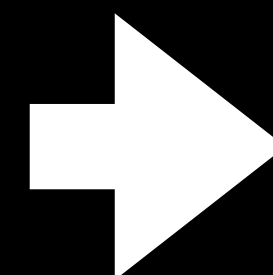


Transformers



Paper text trajectories

Transformers



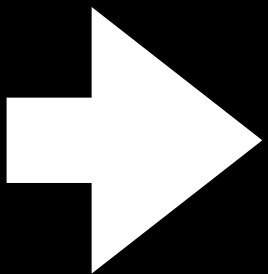
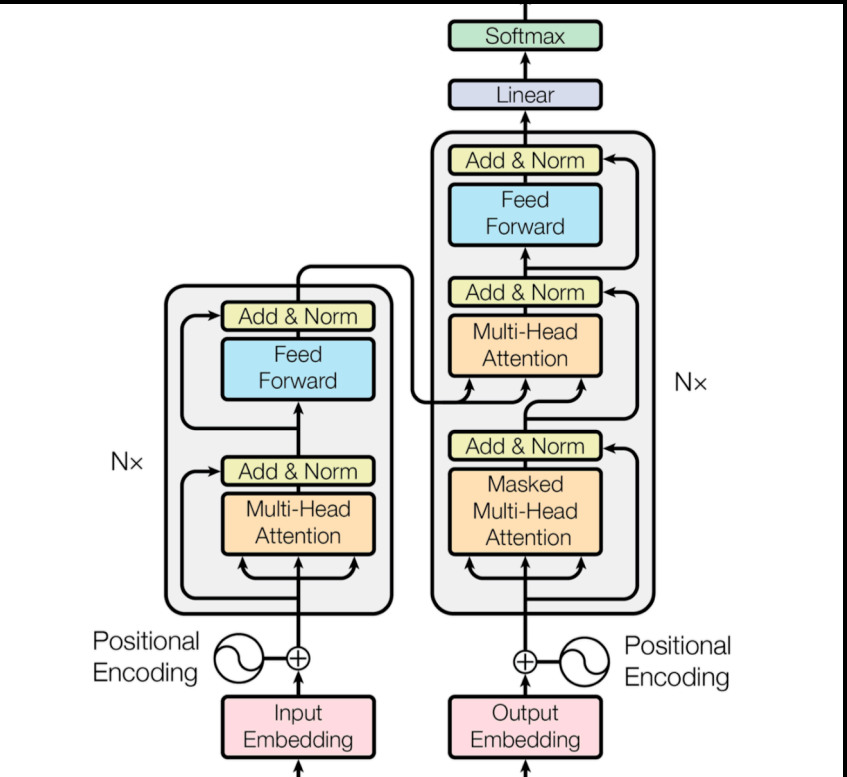
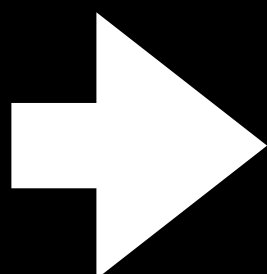
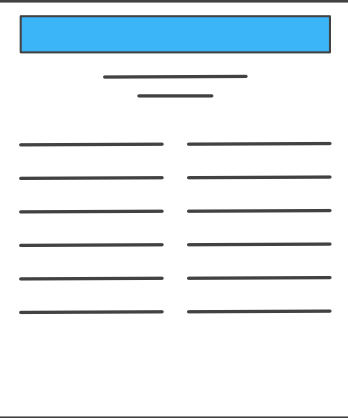
Paper title

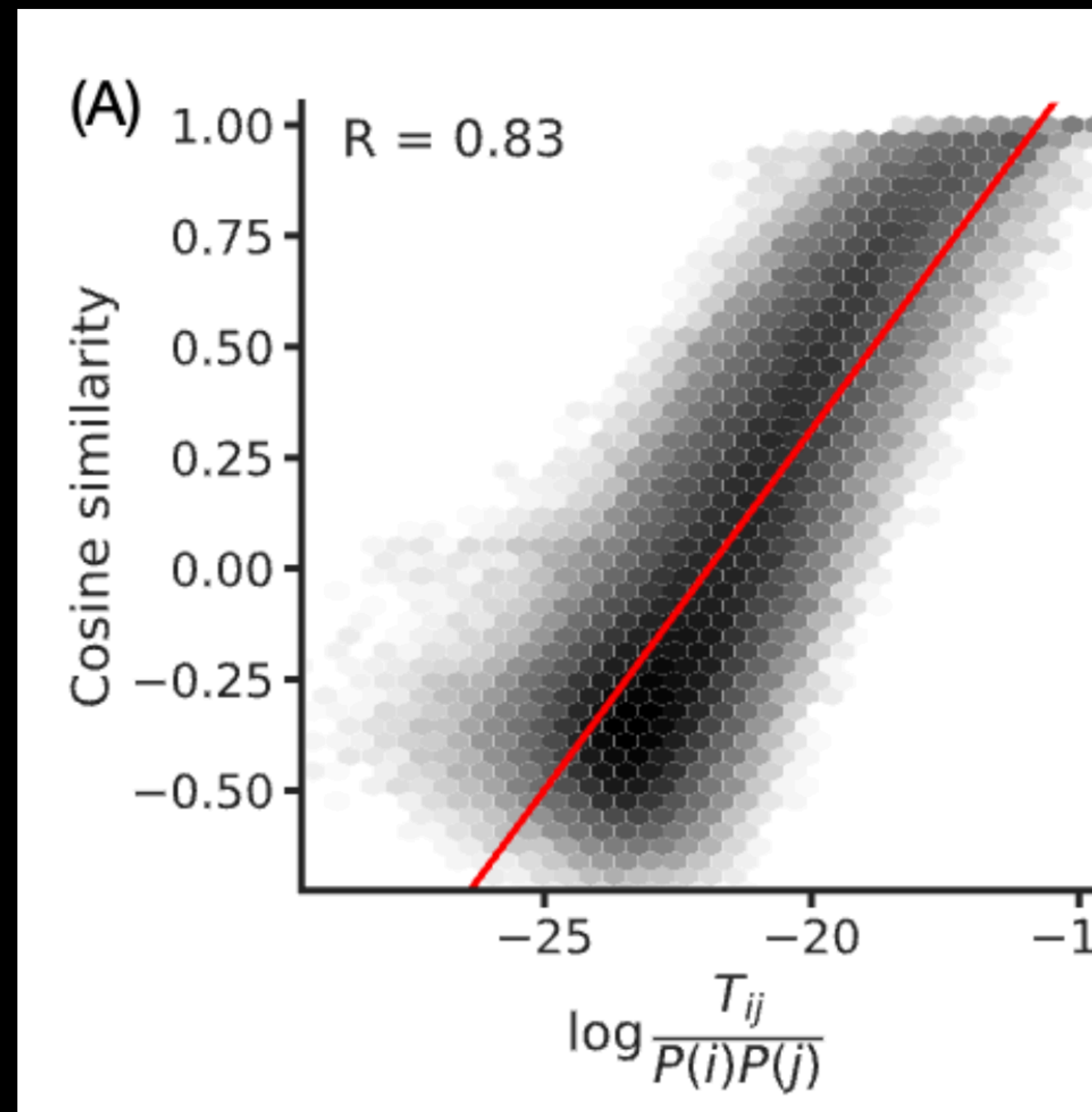


SciSciNet data
(134M papers published
before 2022)

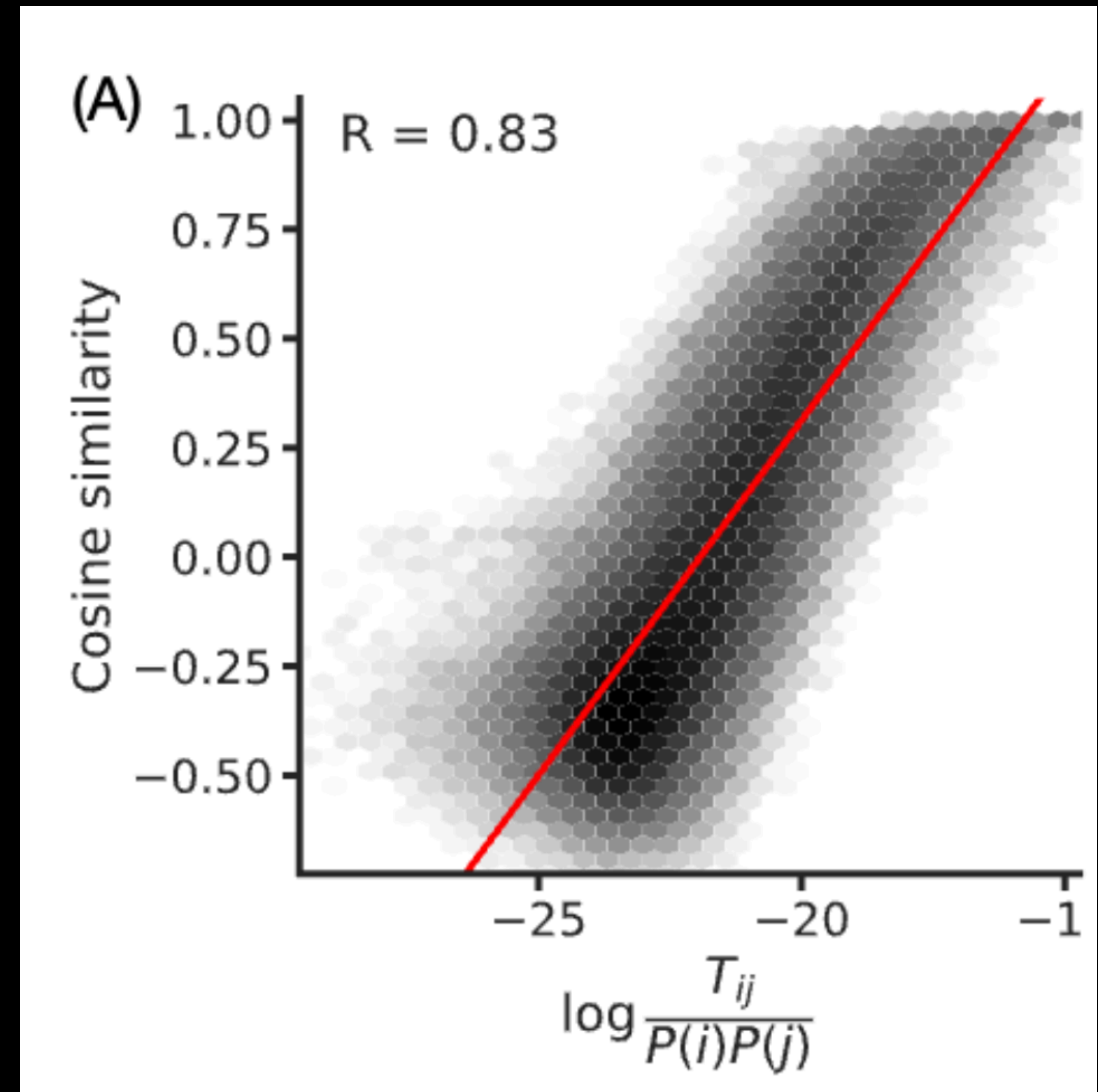
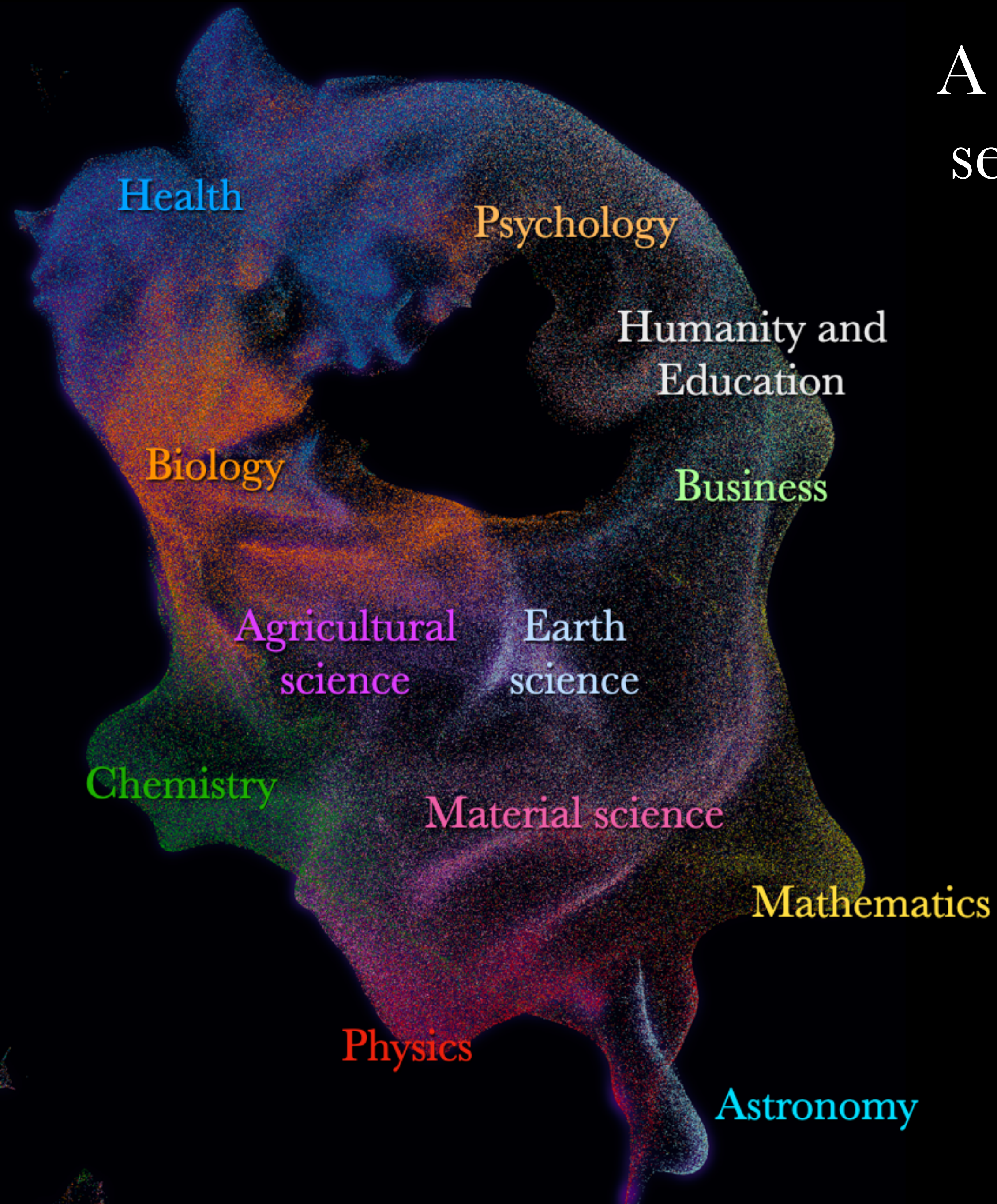
Training

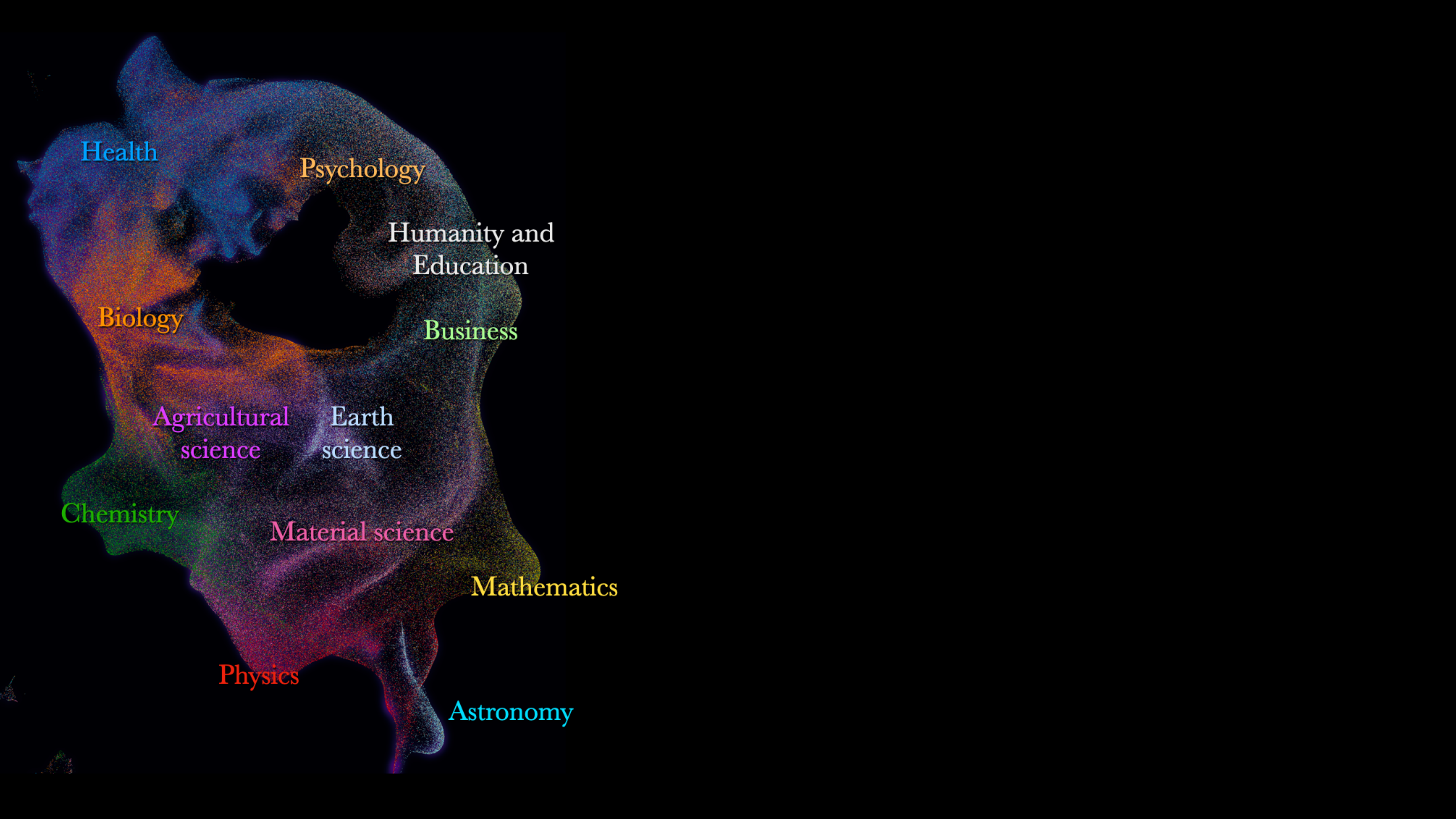
RoBERTa
transformers





A representation space for any possible sentences, where the gravity law holds





Health

Psychology

Humanity and
Education

Biology

Business

Agricultural
science

Earth
science

Chemistry

Material science

Mathematics

Physics

Astronomy



An LLM that produces embeddings of any paper titles, where distance captures *how easy for scientists to make the jump*

We can use intuitive space analogy.



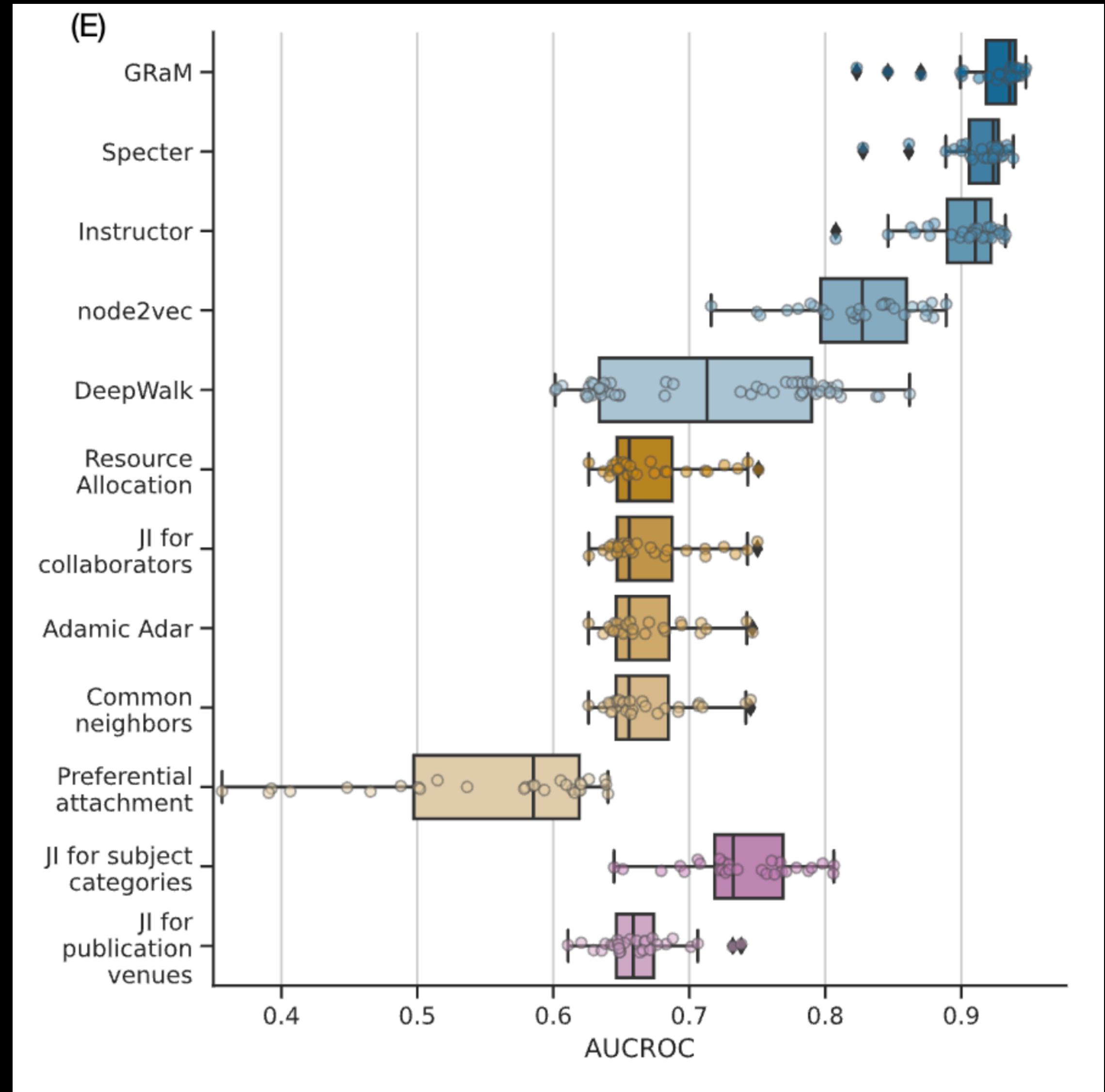
Predicting new collaborations

Pick two scientists who
have never collaborated



Measure the distance
in the given year



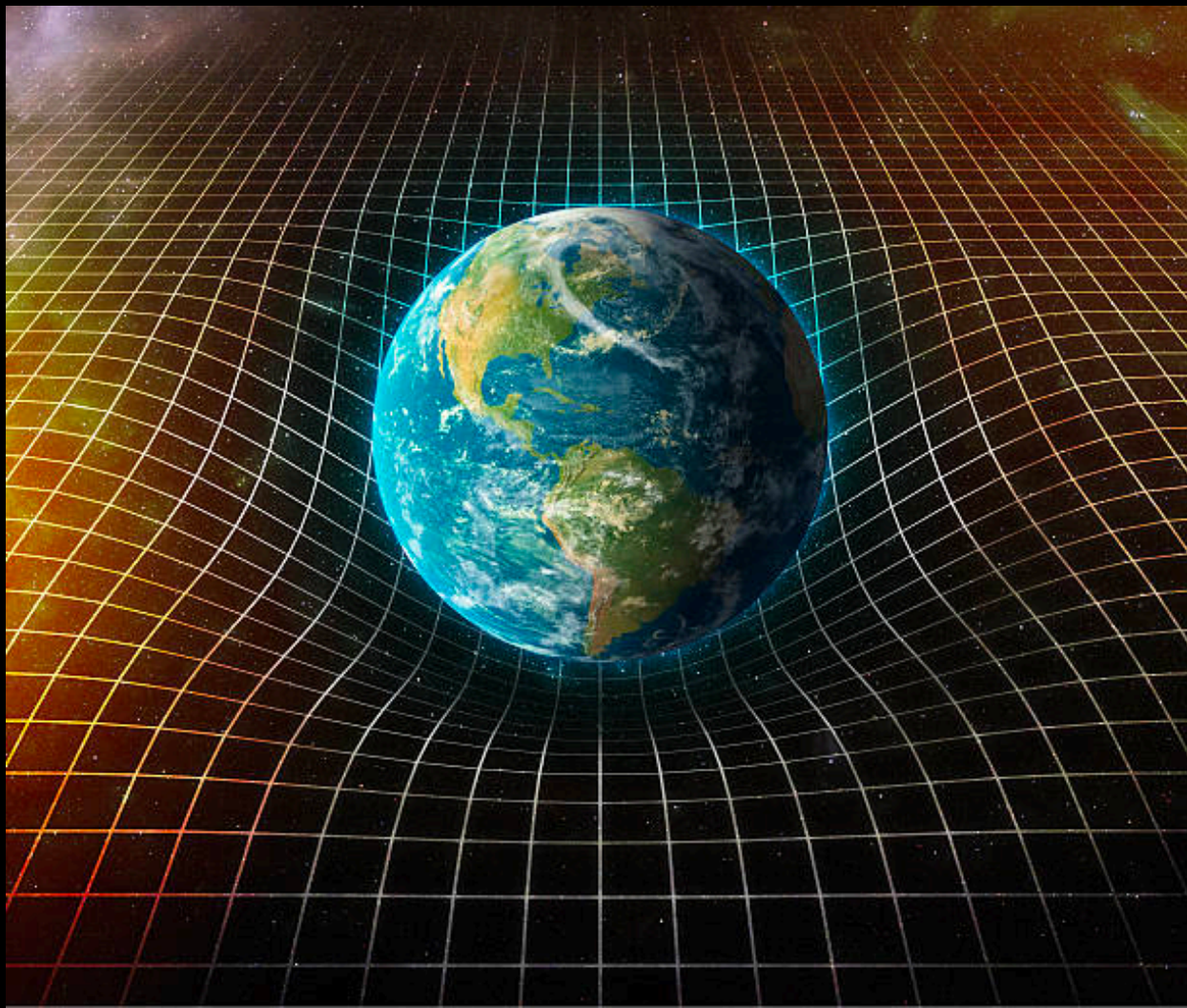


We can bring tools from physics



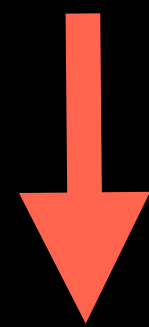
New attraction area

Gravity potential



Gravity law of embedding

$$P(i, j) = \frac{1}{Z} P(i) P(j) \exp(v_j^\top u_i)$$



Gravity potential

$$\Phi(u) = -\frac{1}{Z} \sum_j P(j) \exp(v_j^\top u)$$

Gravity potential
operationalizes how
many scientists an
area attracts

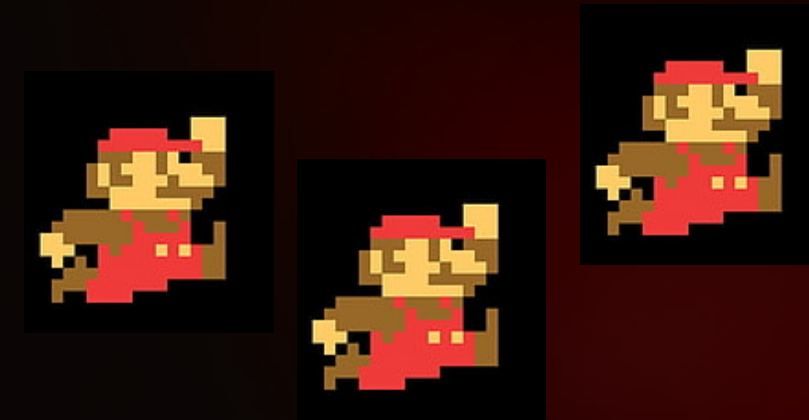
Computer Science

Physics

Social
Sciences



Computer Science



Science of
Science



Physics

Social
Sciences



Computer Science



Physics

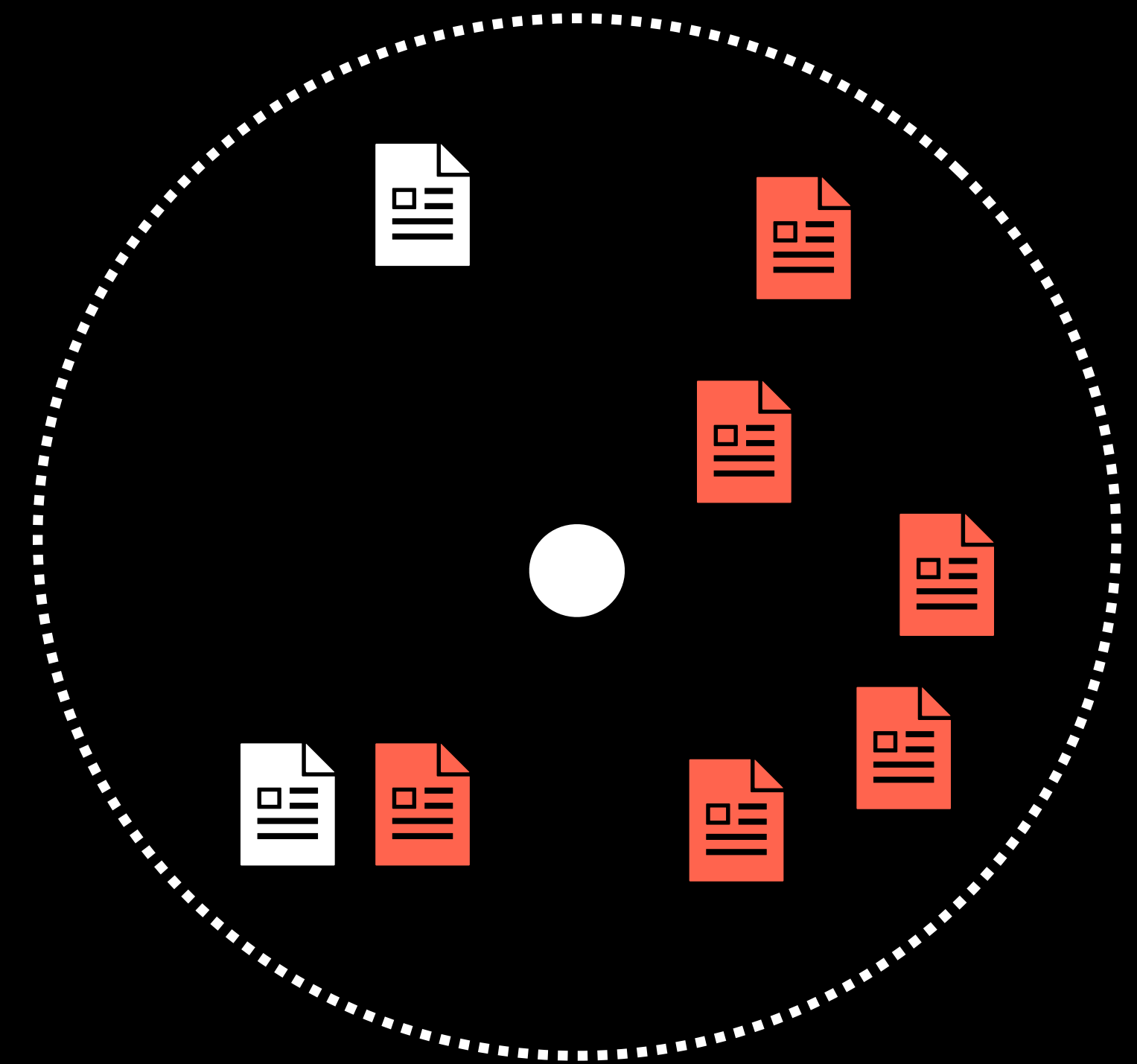
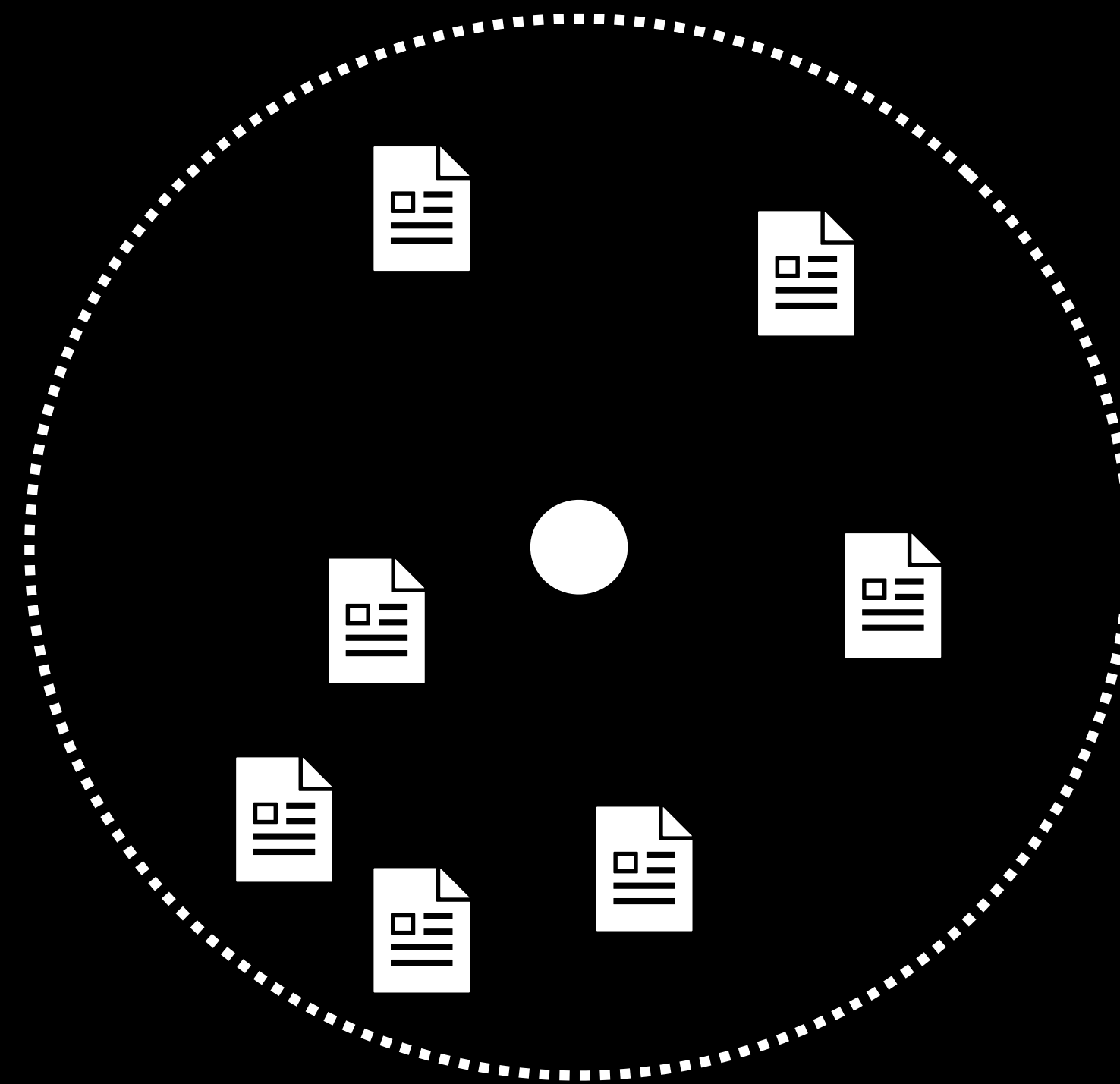
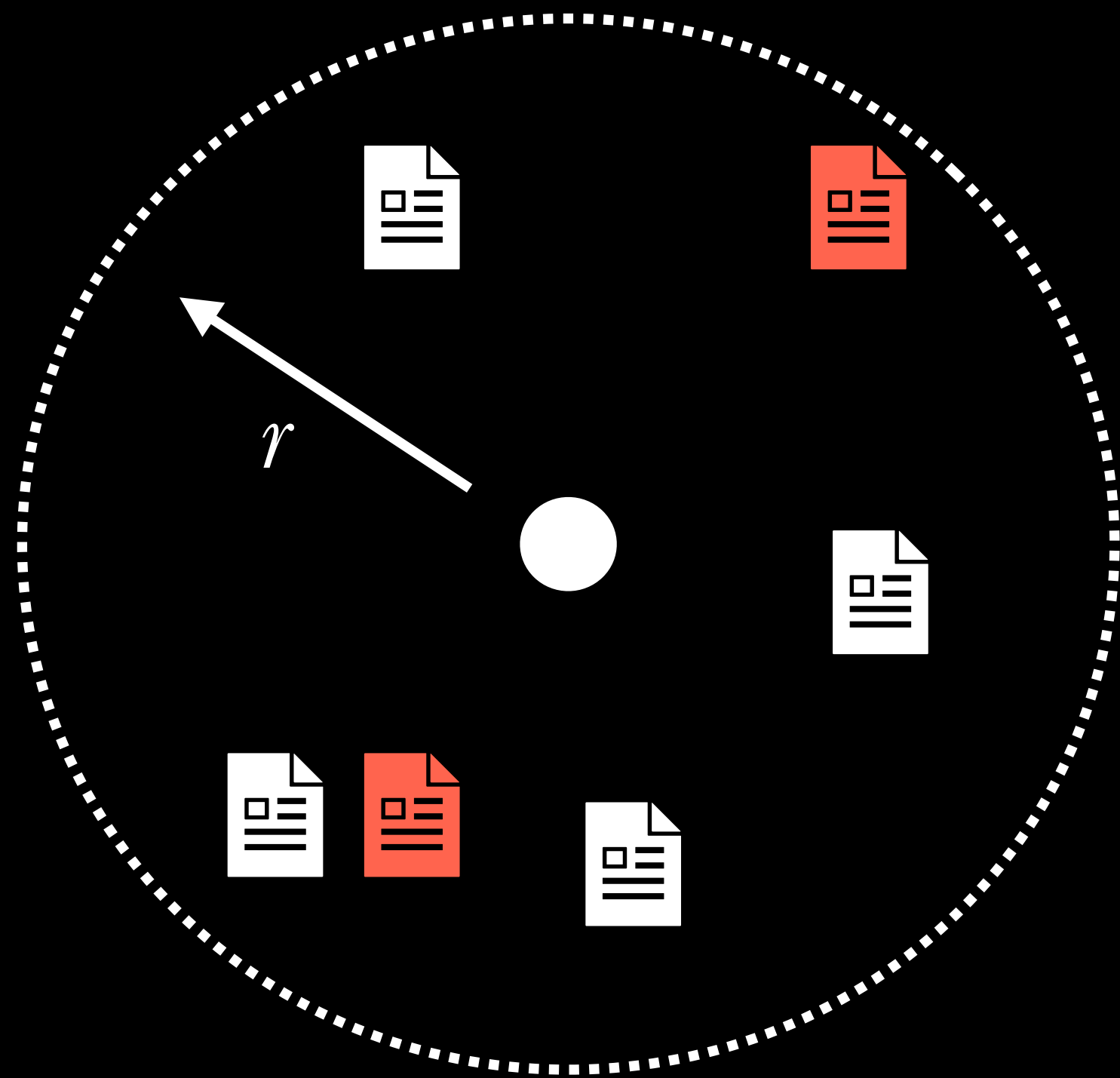
Science of
Science



Social
Sciences

*Emerging fields
~ black holes?*

Can our gravity potential predict new journals?

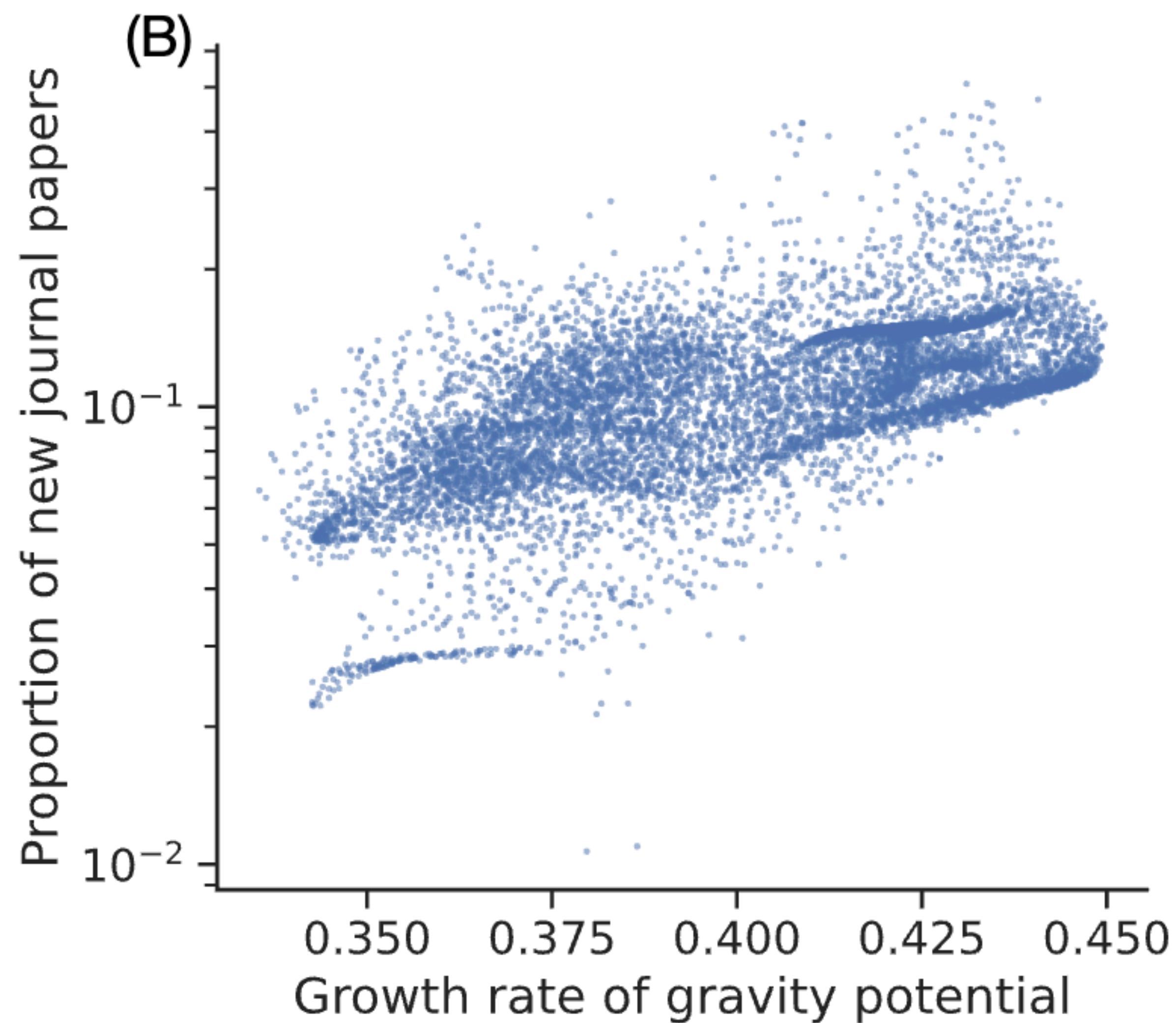
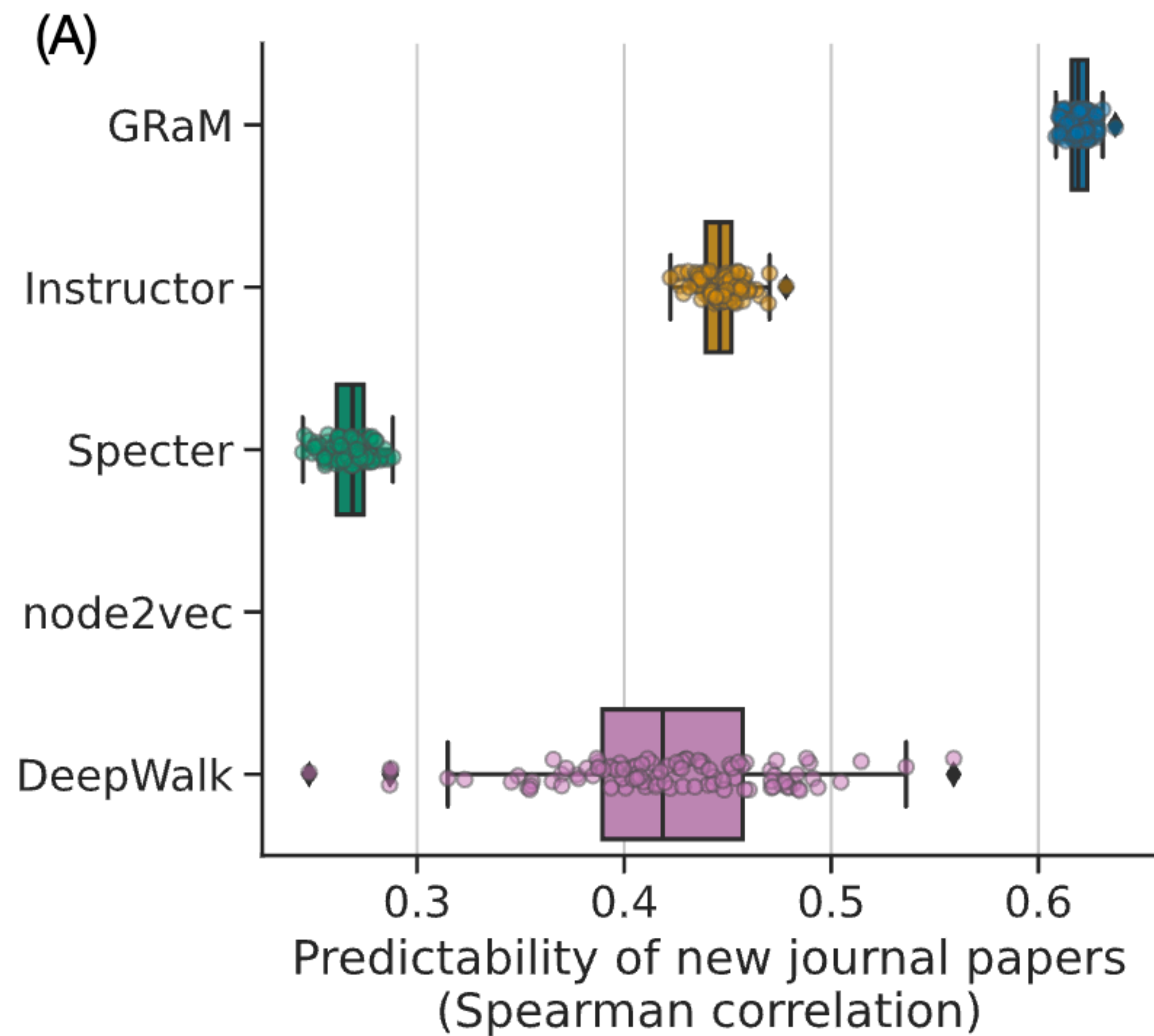


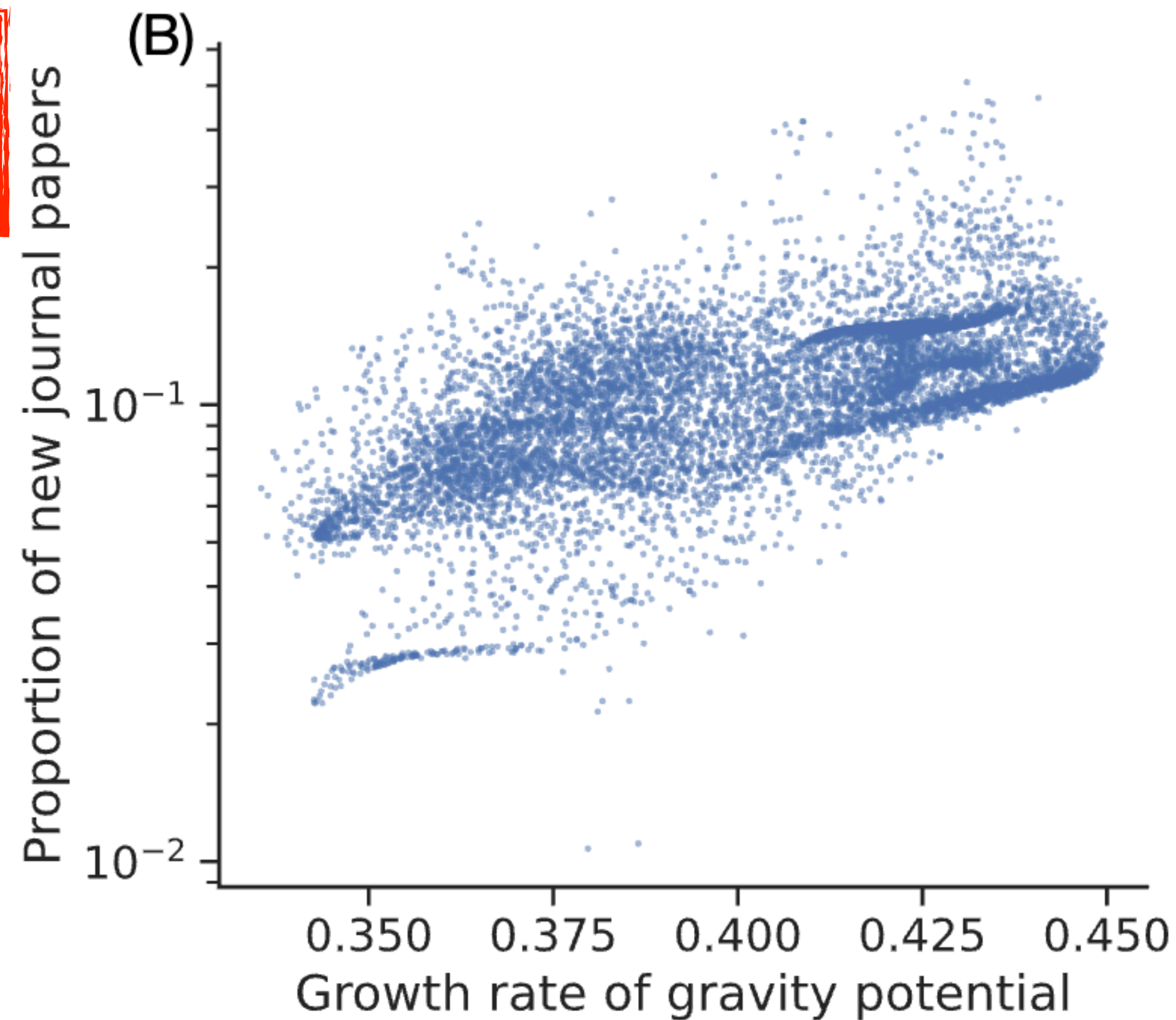
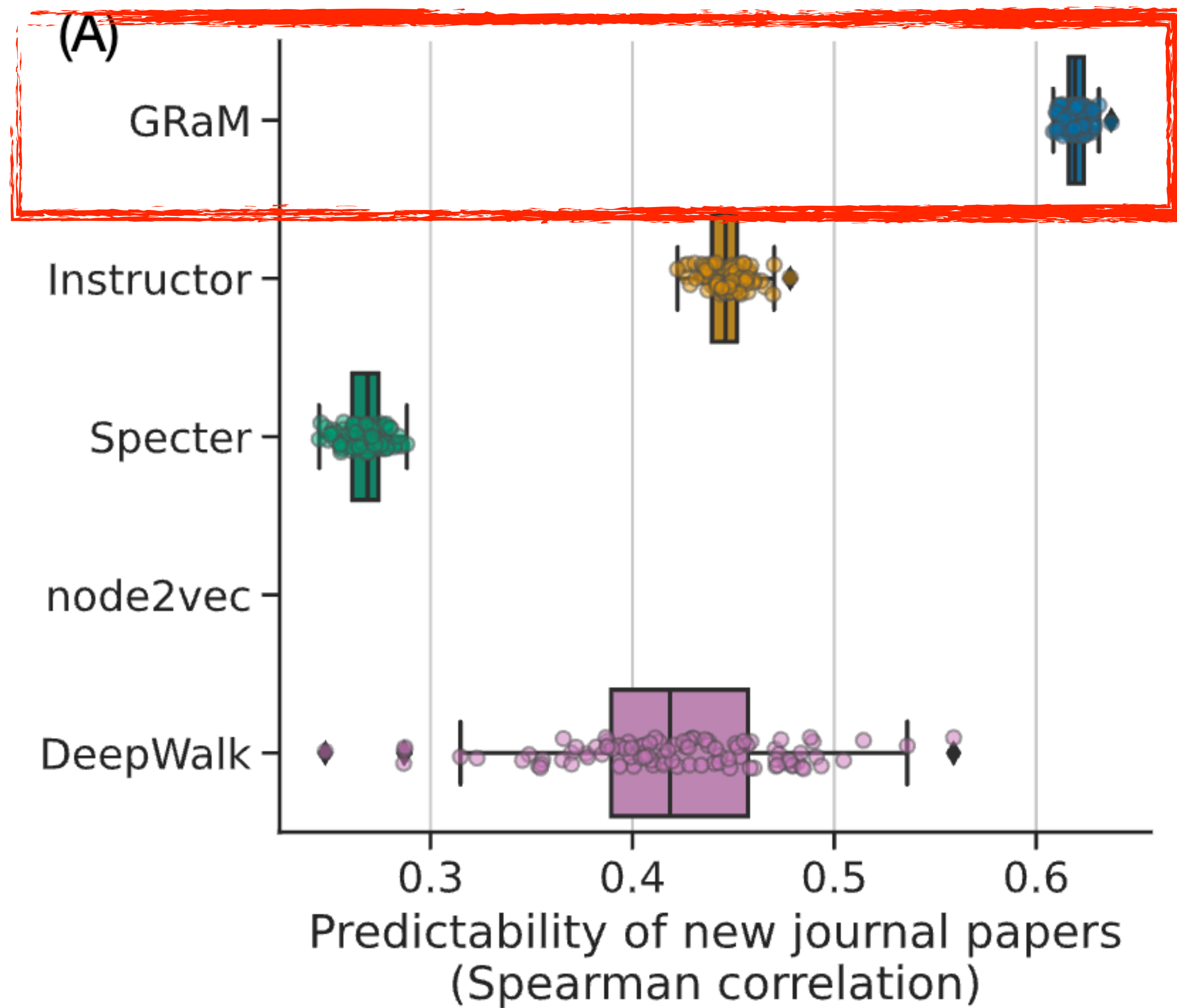
Our prediction score

$$\log \frac{\Phi_{[2006,2010]}(u)}{\Phi_{[2000,2005]}(u)}$$

Evaluation metric $\sim$

Proportion of **new journal papers** within a radius r

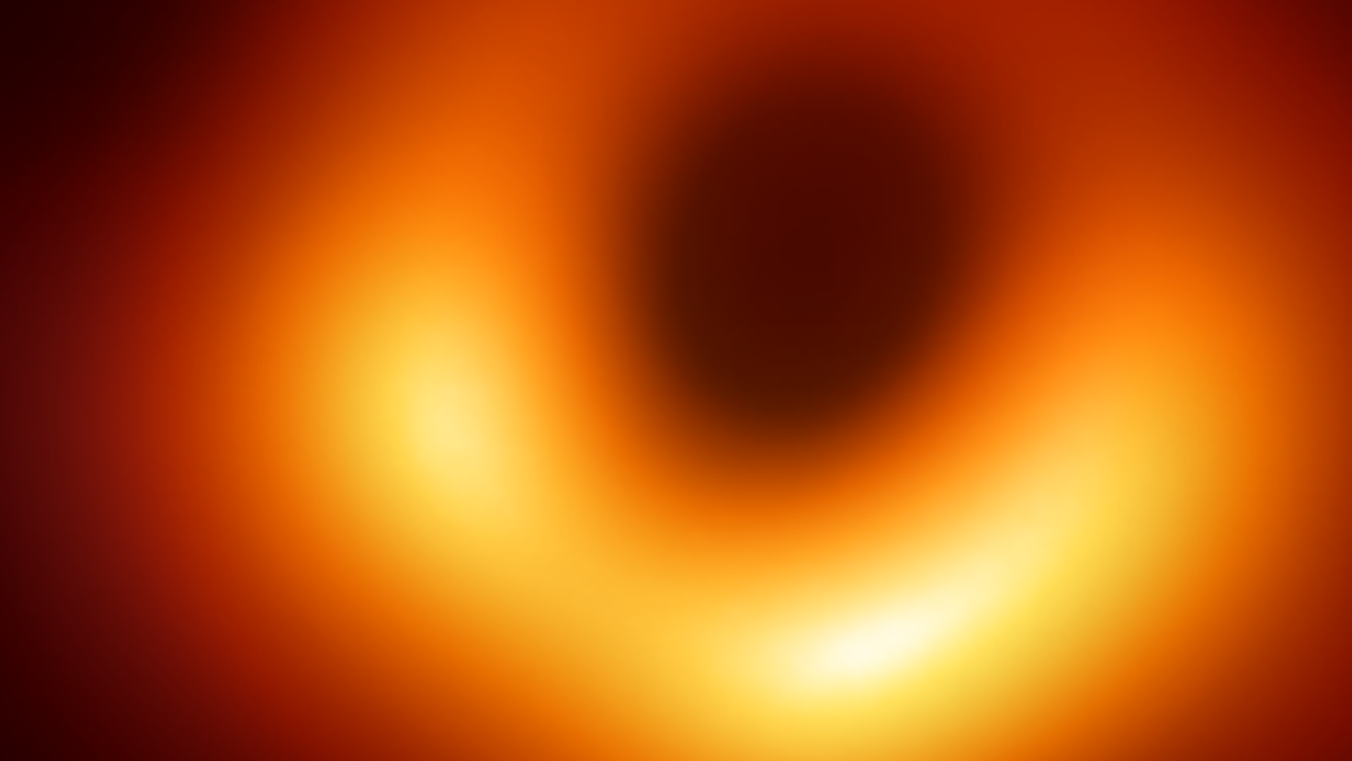




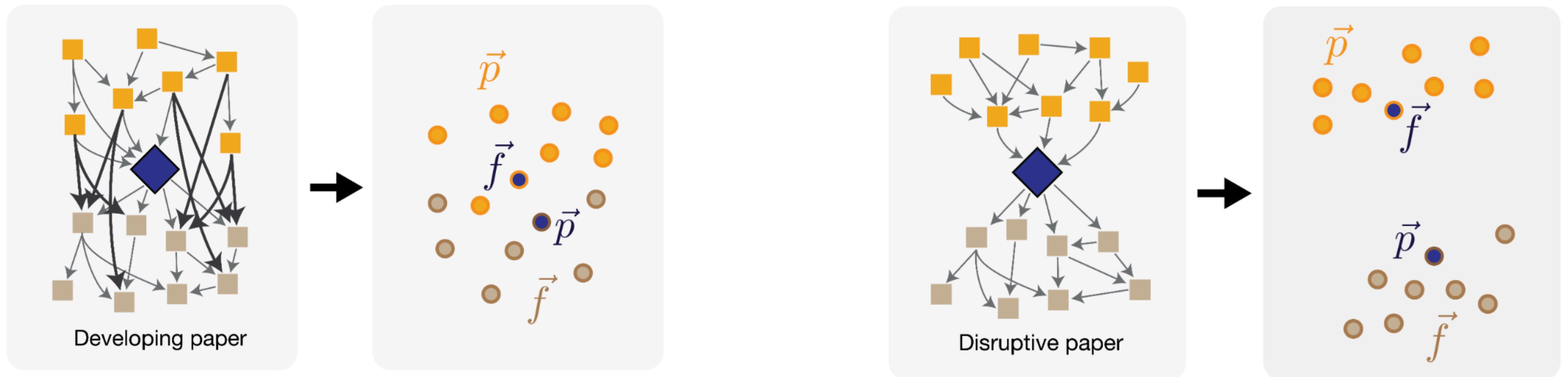
Summary

- Understanding of embedding model → Interpretable embedding space.
- Transition probability and gravity law can guide the creation of interpretable knowledge space models.

**Thinking disruptiveness in knowledge
space**

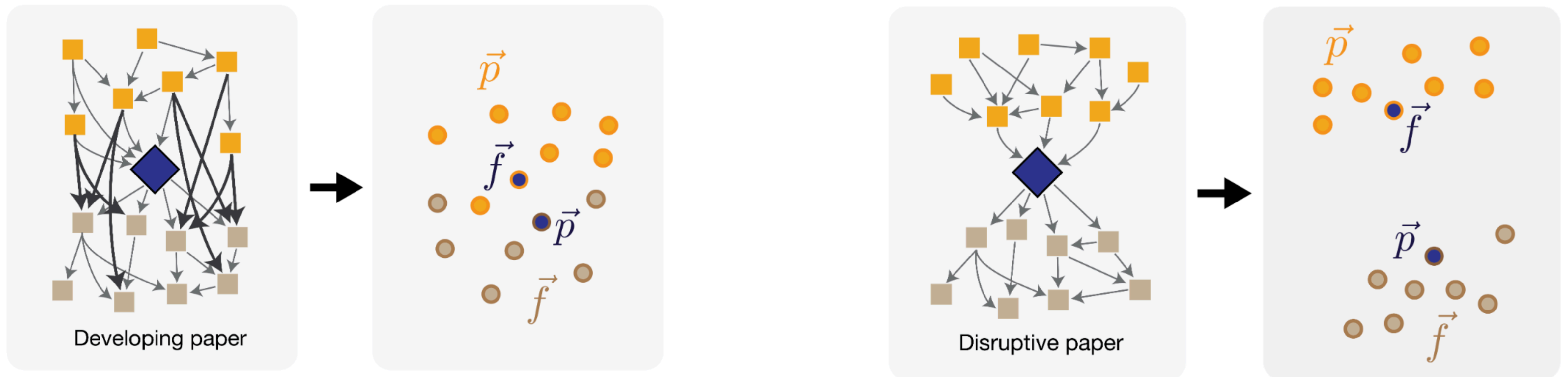


Imagining the knowledge space allows us to define a powerful continuous measure of disruption.



Check out the poster!

Imagining the knowledge space allows us to define a powerful continuous measure of disruption.



Summary

- Imagining the space of knowledge help us conceptualize:
 - A continuous measure of disruptiveness
 - A community-centric model of long term citation patterns
- We can create a representation model of knowledge space that can be concretely interpretable with the gravity law.



Staša Milojević,
IU



Alessandro Flammini,
IU



Filippo Menczer,
IU



Sriraam Natarajan,
UT Dallas



Hao Peng



Sadamori Kojaku



Robert Mahari



Sandro Lera



Sandy Pentland



Clara Boothby



Esteban Moro



Rodrigo Costas



Ceren Budak



Daniel Romero



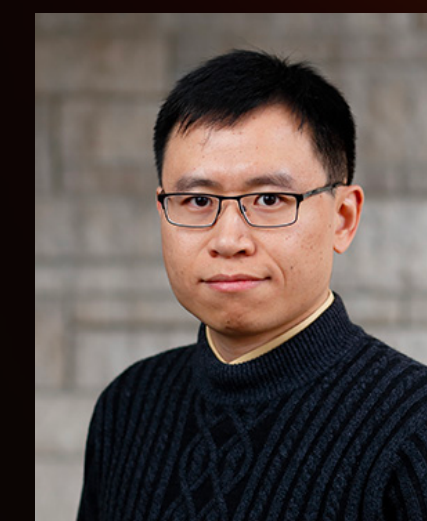
Filipi Silva



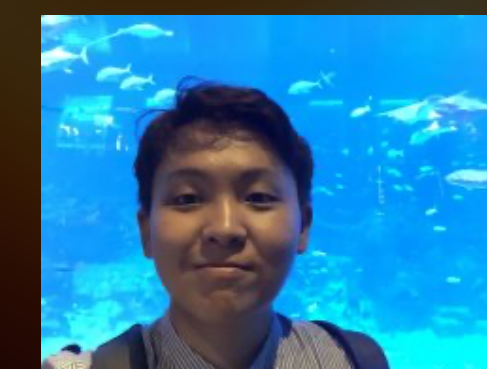
Lili Miao



Dakota Murray



Xiaoran Yan



Jisung Yoon



Woo-Sung Jung



Cassidy Sugimoto



Vincent Lariviere



Attila Varga

Thanks!